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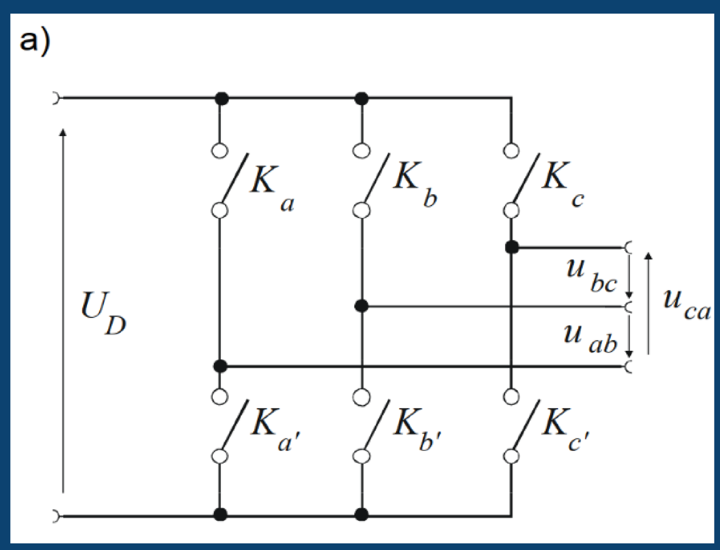
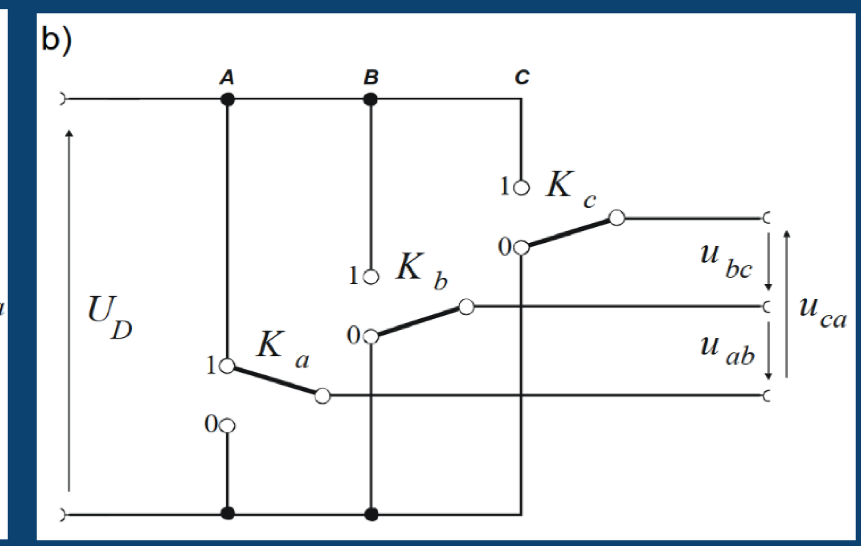
Jan Iwaszkiewicz, Adam Muc

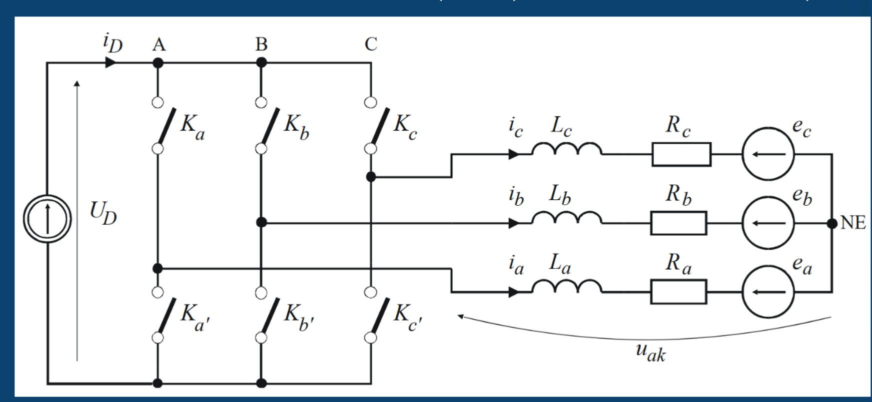
An analytic model of the CSI

The three-phase inverter represents a complex structure consisted of ideal two-state switches. The substitute scheme of the three-phase inverter is presented in Fig below. The inverter is built from three branches: A, B, C, which are assigned to particular phases. The corresponding quantities and variables are categorized as: a, b, c respectively. Two most popular versions of the three-phase inverter models are presented in Figures. The Figure a) presents a circuit constructed of six switches: Ka, Ka', Kb, Kb', Kc, Kc', whereas in Figure b) a circuit built of three switches: Ka, Kb, Kc is shown. Load is connected to three inverter outputs: Swa, Swb, Swc.





The circuit with six switches imitates the 'physics' of the two-level three-phase inverter. It is necessary to note that only such a construction allows estimation of all states of the three-phase inverter. The simultaneously switching two switches on or off in one branch can only be assessed by the model in Fig. a). A simpler in description model in Figure b) proved to be more useful for analyzing voltage source inverters (VSI), what resulted in common application and practice. The object of the paper is to analyze the current source inverters (CSI) so, the model a) will be well thought-out.



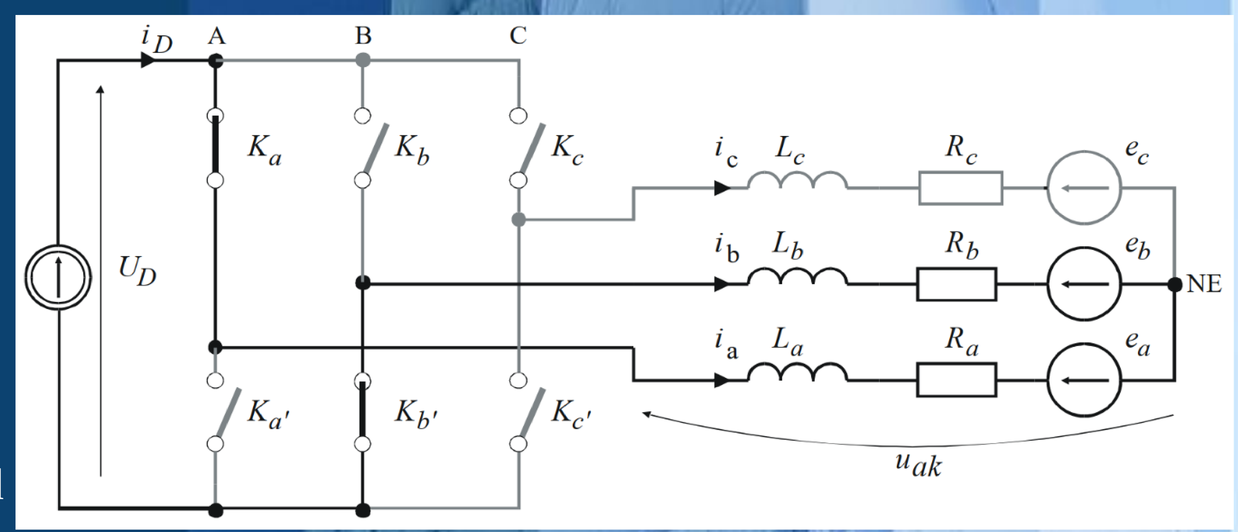
Current Source Inverter in Time Domain –Selected States

Decimal numbers (0 to 63) are the decimal equivalents of binary numbers. They depict the subsequent current states in determined inverter phases. These states are called the CSI vectors and are labelled with the symbol. The vector is defined as the three elements row of a matrix $\vec{I}_k^t = \{i_{ak}, i_{bk}, i_{ck}\}$ for $k = 0, 1, 2, 3, \ldots, 63$, where phases currents assume the values presented in Table. Table contains only such active and zero vectors , which occur in abnormal or fault-free operational states of the CSI. The introduced definition of the current vector has to be applied in the time domain because it operates on physical currents.

Table Phases currents i_{ak}, i_{bk}, i_{ck} for selected current inverter vectors.

$\vec{I}_k^t$	$\vec{I}_3^t$	$\vec{I}_{12}^t$	$\vec{I}_{48}^t$	$\vec{I}_6^t$	$\vec{I}_9^t$	$\vec{I}_{18}^t$	$\vec{I}_{24}^t$	$\vec{I}_{33}^t$	$\vec{I}_{36}^t$
i_{ak}	0	0	0	0	0	$-I_D$	$-I_D$	I_D	I_D
i_{bk}	0	0	0	$-I_D$	I_D	0	I_D	0	$-I_D$
i_{ck}	0	0	0	I_D	$-I_D$	I_D	0	$-I_D$	0

The substitute system of the three-phase CSI is presented in Figure on the right. The parameter which distinguishes the inverter is the constant value of the intermediary circuit current ID. The discussed inverter load has been connected in a star. The switches Ka, Ka', Kb, Kb', Kc, Kc' depending on the selected current vector, connect the intermediary circuit current ID to selected load branches. It is assumed that there is an ideal source of direct current ID in the intermediary circuit, as well as phases load is symmetrical. For any current vector $\vec{I}_k^t$ starting at the time point t = Tn the three-phase inverter model is determined by one closed loop substitute circuit. It was assumed that the phase ***c*** load has not been connected to the current source.



For all vectors evaluated in Table phase voltages are expressed by the following equations:

$$\left.\begin{aligned}
u_{a_k}(t) &= R_a \cdot I_D + L_a \frac{dI_D}{dt} + e_a(t)\ \textbf{for}\ k = 33,36\\
u_{a_k}(t) &= -R_a \cdot I_D - L_a \frac{dI_D}{dt} + e_a(t)\ \textbf{for}\ k = 18,24\\
u_{a_k}(t) &= e_a(t)\ \textbf{for}\ k = 6,9
\end{aligned}\right\}$$

$$\left.\begin{aligned}
u_{b_k}(t) &= R_b \cdot I_D + L_b \frac{dI_D}{dt} + e_b(t)\ \textbf{for}\ k = 9,24\\
u_{b_k}(t) &= -R_b \cdot I_D - L_b \frac{dI_D}{dt} + e_b(t)\ \textbf{for}\ k = 6,36\\
u_{b_k}(t) &= e_b(t)\ \textbf{for}\ k = 18,33
\end{aligned}\right\}$$

$$\left.\begin{aligned}
u_{c_k}(t) &= R_c \cdot I_D + L_c \frac{dI_D}{dt} + e_c(t)\ \textbf{for}\ k = 6,18\\
u_{c_k}(t) &= -R_c \cdot I_D - L_c \frac{dI_D}{dt} + e_c(t)\ \textbf{for}\ k = 9,33\\
u_{c_k}(t) &= e_c(t)\ \textbf{for}\ k = 24,36
\end{aligned}\right\}$$

In the ideal model the switch on and off times equal ton = 0 and toff = 0. As a consequence, in equation solutions and results Dirac's delta function appears. This function is labelled with Dirac's delta while the expressions of phase voltage take the form of dependencies:

Phase a voltage:

$$\left.\begin{aligned}
u_{a_k}(t) &= R_a \cdot I_D + \eta \cdot \sigma(t - T_n) + e_a(T_n)\ \textbf{for}\ k = 33,36\\
u_{a_k}(t) &= -R_a \cdot I_D - \eta \cdot \sigma(t - T_n) + e_a(T_n)\ \textbf{for}\ k = 18,24\\
u_{a_k}(t) &= e_a(T_n)\ \textbf{for}\ 6,9
\end{aligned}\right\}\quad for\ t = T_n$$

$$\left.\begin{aligned}
u_{a_k}(t) &= R_a \cdot I_D + e_a(t)\ \textbf{for}\ k = 33,36\\
u_{a_k}(t) &= -R_a \cdot I_D + e_a(t)\ \textbf{for}\ k = 18,24\\
u_{a_k}(t) &= e_a(t)\ \textbf{for}\ k = 6,9
\end{aligned}\right\}\quad for\ T_n < t < T_{n+1}$$

$$\left.\begin{aligned}
u_{a_k}(t) &= R_a \cdot I_D + \eta \cdot \sigma(t - T_{n+1}) + e_a(T_{n+1})\ \textbf{for}\ k = 33,36\\
u_{a_k}(t) &= -R_a \cdot I_D - \eta \cdot \sigma(t - T_{n+1}) + e_a(T_{n+1})\ \textbf{for}\ k = 18,24\\
u_{a_k}(t) &= e_a(T_{n+1})\ \textbf{for}\ 6,9
\end{aligned}\right\}\quad for\ t = T_{n+1}$$

Phase b voltage:

$$\left.\begin{aligned}
u_{b_k}(t) &= R_b \cdot I_D + \eta \cdot \sigma(t - T_n) + e_b(T_n)\ \textbf{for}\ k = 9,24\\
u_{b_k}(t) &= -R_b \cdot I_D - \eta \cdot \sigma(t - T_n) + e_b(T_n)\ \textbf{for}\ k = 6,36\\
u_{b_k}(t) &= e_b(T_n)\ \textbf{for}\ k = 18,33
\end{aligned}\right\}\quad for\ t = T_n$$

$$\left.\begin{aligned}
u_{b_k}(t) &= R_b \cdot I_D + e_b(t)\ \textbf{for}\ k = 9,24\\
u_{b_k}(t) &= -R_b \cdot I_D + e_b(t)\ \textbf{for}\ k = 6,36\\
u_{b_k}(t) &= e_b(t)\ \textbf{for}\ k = 18,33
\end{aligned}\right\}\quad for\ T_n < t < T_{n+1}$$

$$\left.\begin{aligned}
u_{b_k}(t) &= R_b \cdot I_D + \eta \cdot \sigma(t - T_{n+1}) + e_b(T_{n+1})\ \textbf{for}\ k = 9,24\\
u_{b_k}(t) &= -R_b \cdot I_D - \eta \cdot \sigma(t - T_{n+1}) + e_b(T_{n+1})\ \textbf{for}\ k = 6,36\\
u_{b_k}(t) &= e_b(T_{n+1})\ \textbf{for}\ k = 18,33
\end{aligned}\right\}\quad for\ t = T_{n+1}$$

The coefficient η in expressions is the product of $L_{a,b,c}$ and I_D has dimension [V]. Function Dirac's delta replicates an infinite overvoltage when the current source is switched on to the inductive load. The current I_D is switched on, flowing through R, L, SEM, the load and switched off. This process can be described taking into account the following assumptions:

-the considered $T_n < t < T_{n+1}$ time interval load and switched off. This process can be described taking into account the following assumptions: between consequent $\vec{I}_{k(n)}^t$ vector switching. The current flows through the R, L nad e_f laod;

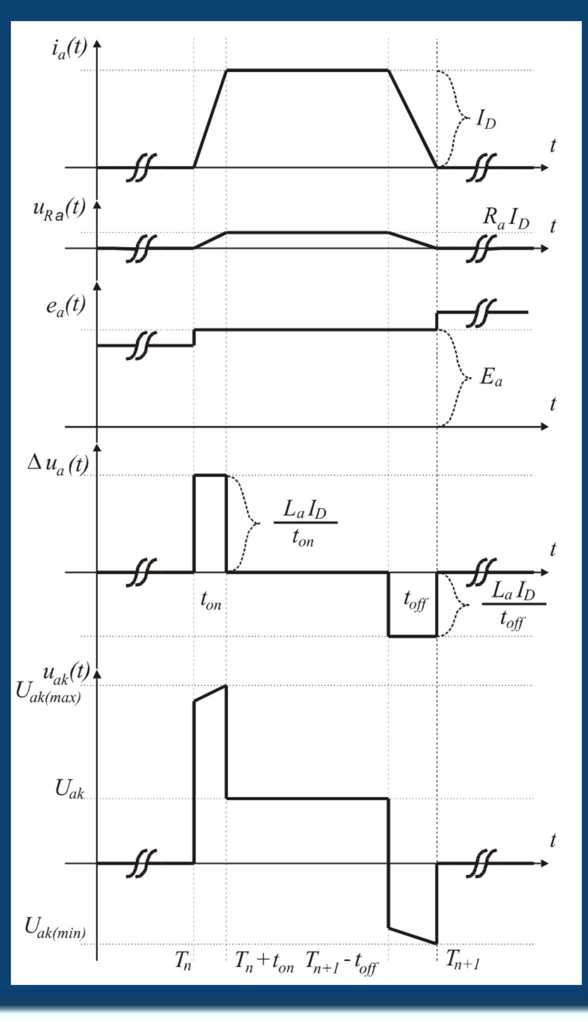
-the current increases linearly from the $i(T_n) = 0$ value to the $i(T_n + t_{on}) = I_D$ value;

-in the $T_n + t_{on} < t < T_{n+1} - t_{off}$ time interval the maintained current value is $i(t) = I_D$;

-the turn off process starts at the time point $t = T_{n+1} - t_{off}$ and takes place in the $t = t_{off}$ time, which is called the turn off time,

-the current decreases linearly from the $i(T_{n+1} + t_{off}) = I_D$ value to the $i(T_{n+1}) = 0$;

Taking into consideration aforementioned assumptions, equations' solutions describe the phases voltage waveforms in consequent time periods: $T\!: < T_n, T_n + t_{on} >$, $< T_n + t_{on}, T_{n+1} - t_{off} >$ and $< T_{n+1} - t_{off}, T_{n+1} >$.



For example, the solutions for phase a and consequent vectors take the form of expressions:

$$\left.\begin{aligned}
u_{a_k}(t) &= R_a \cdot I_D + \frac{L_a \cdot I_D}{t_{on}} + e_a(t)\ \textbf{for}\ k = 33,36\\
u_{a_k}(t) &= -R_a \cdot I_D - \frac{L_a \cdot I_D}{t_{on}} + e_a(t)\ \textbf{for}\ k = 18,24\\
u_{a_k}(t) &= e_a(t)\ \textbf{for}\ 6,9
\end{aligned}\right\}\quad T_n \le t \le T_n + t_{on}$$

$$\left.\begin{aligned}
u_{a_k}(t) &= R_a \cdot I_D + e_a(t)\ \textbf{for}\ k = 33,36\\
u_{a_k}(t) &= -R_a \cdot I_D + e_a(t)\ \textbf{for}\ k = 18,24\\
u_{a_k}(t) &= e_a(t)\ \textbf{for}\ k = 6,9
\end{aligned}\right\}\quad T_n + t_{on} < t < T_{n+1} - t_{off}$$

$$\left.\begin{aligned}
u_{a_k}(t) &= R_a \cdot I_D + \frac{L_a \cdot I_D}{t_{off}} + e_a(t)\ \textbf{for}\ k = 33,36\\
u_{a_k}(t) &= -R_a \cdot I_D - \frac{L_a \cdot I_D}{t_{off}} + e_a(t)\ \textbf{for}\ k = 18,24\\
u_{a_{k(t)}} &= e_a(t)\ \textbf{for}\ 6,9
\end{aligned}\right\}\quad T_{n+1} - t_{off} \le t \le T_{n+1}$$

The same way permits to obtain phase-to-phase voltage for consecutive current vectors as well as phase-to-phase voltage u_{bc} and u_{ca}.

The Intermediary Circuit Voltage:

The intermediary circuit voltage is easily determined by use of phase-to-phase voltages, since in the current inverter, regardless of the load configuration, voltage U_D is always correspondent to one phase-to-phase voltage.

For the next current vectors, voltage U_D is calculated from the phase-to-phase voltage displayed in Table:

Table The intermediary circuit voltage for certain current vectors

$\vec{I}_k^t$	$\vec{I}_3^t$	$\vec{I}_{12}^t$	$\vec{I}_{48}^t$	$\vec{I}_6^t$	$\vec{I}_9^t$	$\vec{I}_{18}^t$	$\vec{I}_{24}^t$	$\vec{I}_{33}^t$	$\vec{I}_{36}^t$
U_D	0	0	0	$-U_{bc}$	U_{bc}	U_{ca}	$-U_{ab}$	$-U_{ca}$	U_{ab}

A Unified Current Inverter Model with Load Connected in a Star (MFPG)

To describe the CSIM (CSI Model), understanding of the intermediary circuit current, the selected current vector, time and moment of starting as well as load parameters are fundamental.
In the brace bracket all the essential independent variables have been placed.

$$CSIM = \left\{I_D, \vec{I}_k^t, Z_f, E_f, T_n \le t \le T_{n+1}\right\}$$

If in the ideal model of the current inverter the real turn on and off times of switches are considered, the phases voltages $u_{ak}(t)$, while phases voltages $u_{bk}(t)$ and $u_{ck}(t)$ – progressing analogously to the procedures described for phase a.

CONCLUSION

The paper presents a very simple analytic model of the CSI as well as a mathematical system of notation and formulas. The current states of the current source inverter may be defined by use of definite state vectors. The equations describing voltage and current are easy-to-use mathematical tools. They permit to select suitable vector sequence assuring the desirable voltage and current waveform. Thus, they facilitate to design control algorithms of CSIs and particularly current control of AC drives. This mathematical tool was verified during simulation and experimental tests. The considered method could be advanced to other multiphase and multilevel current inverters.