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Three-phase Quaternion Power in Three-wire Systems

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Abstract

Instantaneous power theory has a central role in power systems analysis. Among mathematical settings used for the development of this theory, quaternion algebra has been used for describing electrical variables in recent works. In this context, this paper aims to describe three-phase power in a quaternion framework. We analyze quaternion power for balanced and unbalanced delta loads, comparing the expressions obtained to the usual expressions of complex power. The quaternion power expression obtained also makes it natural to introduce a decomposition of the unbalanced load in terms of a balanced component and an unbalanced load with null average power. It is also shown that delta unbalanced loads are equivalent to time-varying balanced loads. The results obtained extend the power systems theory in the quaternion domain and emphasize the advantages of using this framework.

Introduction

- A framework based on quaternions seems to be promising for analysis of several problems in power systems and has already produced some interesting results.
- In this work, we study the quaternionic power for a general load in Δ configuration.

Quaternion Definition and Properties

- Quaternions can be defined as

$$\boldsymbol{A} = r + x\boldsymbol{q_1} + y\boldsymbol{q_2} + z\boldsymbol{q_3}$$

where r, x, y and z are real numbers and 1, $\boldsymbol{q_1}$, $\boldsymbol{q_2}$ and $\boldsymbol{q_3}$ form the orthonormal basis of the hypercomplex space.

- The product of two quaternions is non-commutative.

Table I – Product of basic quaternions

	1	q_1	q_2	q_3
1	1	q_1	q_2	q_3
q_1	q_1	-1	q_3	$-q_2$
q_2	q_2	$-q_3$	-1	q_1
q_3	q_3	q_2	$-q_1$	-1

Quaternion Power in Three-wire Systems

- The quaternion balanced voltage can be defined by

$$\boldsymbol{V}(t) = v_a(t)\boldsymbol{q_1} + v_b(t)\boldsymbol{q_2} + v_c(t)\boldsymbol{q_3}$$

where $v_a(t) = V_o\cos(\omega t)$, $v_b(t) = V_o\cos(\omega t - \frac{2\pi}{3})$ and $v_c(t) = V_o\cos(\omega t + \frac{2\pi}{3})$.

- Analogously, the three-phase current quaternion are defined by

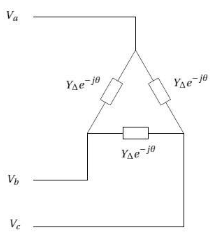
$$\boldsymbol{I}(t) = i_a(t)\boldsymbol{q_1} + i_b(t)\boldsymbol{q_2} + i_c(t)\boldsymbol{q_3}$$

in which $i_a(t)$, $i_b(t)$ and $i_c(t)$ are line currents flowing through phases A, B and C, respectively.

- The power quaternion is defined as

$$\boldsymbol{S} = \boldsymbol{V}(t)\boldsymbol{I}^*(t)$$

A. Three-phase balanced Δ load



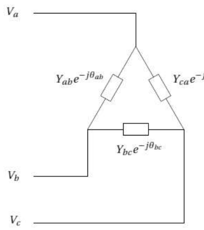
- The quaternion power of the balanced Δ load is

$$\boldsymbol{S} = \frac{9}{2}Y_\Delta V_o^2 e^{\vec{\boldsymbol{n}}\theta}$$

where $\vec{\boldsymbol{n}} = \frac{1}{\sqrt{3}}(\boldsymbol{q_1} + \boldsymbol{q_2} + \boldsymbol{q_3})$.

- The expression of the quaternion power is equivalent to the expression obtained for the complex power of this load, changing the imaginary unit j for the unitary quaternion $\vec{\boldsymbol{n}}$.

B. Three-phase unbalanced Δ load



- The power quaternion of this load can be written as

$$\boldsymbol{S} = \frac{9}{2}\frac{Y_{ab}}{3}V_o^2 e^{\vec{\boldsymbol{n}}\theta_{ab}} + \frac{9}{2}\frac{Y_{bc}}{3}V_o^2 e^{\vec{\boldsymbol{n}}\theta_{bc}} + \frac{9}{2}\frac{Y_{ca}}{3}V_o^2 e^{\vec{\boldsymbol{n}}\theta_{ca}} + \frac{9}{2}\frac{Y_{ab}}{3}V_o^2 e^{\vec{\boldsymbol{n}}\phi_{ab}(t)} + \frac{9}{2}\frac{Y_{bc}}{3}V_o^2 e^{\vec{\boldsymbol{n}}\phi_{bc}(t)} + \frac{9}{2}\frac{Y_{ca}}{3}V_o^2 e^{\vec{\boldsymbol{n}}\phi_{ca}(t)}$$

where $\phi_{ab}(t) = 2\omega t + \frac{\pi}{3} - \theta_{ab}$, $\phi_{bc}(t) = 2\omega t + \pi - \theta_{bc}$ and $\phi_{ca}(t) = 2\omega t - \frac{\pi}{3} - \theta_{ca}$.

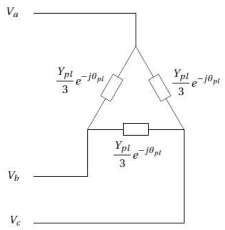
- Comparing the quaternion power to the usual complex power, it can be observed that the three first terms are equivalent to the complex power substituting j for $\vec{\boldsymbol{n}}$.

- The three last terms are unique to the quaternion power representation and account for instantaneous power dynamics.

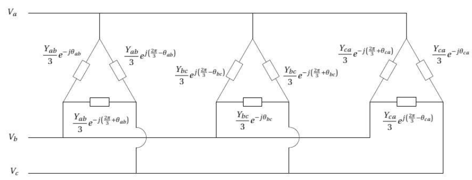
- Summing the three constant quaternions and the three varying in time quaternions, we can rewrite the quaternion power as

$$\boldsymbol{S} = \frac{9}{2}\frac{Y_{pl}}{3}V_o^2 e^{\vec{\boldsymbol{n}}\theta_{pl}} + \frac{9}{2}\frac{Y_\omega}{3}V_o^2 e^{\vec{\boldsymbol{n}}\phi_\omega(t)}$$

- The first term represents the quaternion power of the average of the admittances between each phase.



- The second term is the quaternion power of the association of three loads. These loads have null average power.



- The quaternion power can also be written as

$$\boldsymbol{S} = \frac{9}{2}\frac{Y_\delta(t)}{3}V_o^2 e^{\vec{\boldsymbol{n}}\phi_\delta(t)}$$

- The expression is analogous to the balanced case, implying that the three-phase unbalanced impedance can be represented by a three-phase balanced load, with modulus and phase varying in time.

Conclusion

- The quaternion power can be expressed with two terms:
  - A constant term equivalent to the usual complex power; and
  - A time-varying term that accounts for the contribution of load unbalance on the power.
- The second term can be used as a measure of the load unbalance level.
- An unbalanced Δ load can be decomposed in terms of a balanced load and an unbalanced load with null average power.
- An unbalanced Δ load can be represented as a time-varying balanced load.

References

[1] H. Akagi, Y. Kanazawa, and A. Nabae, “Instantaneous reactive power compensators comprising switching devices without energy storage components,” IEEE Transactions on industry applications, no. 3, pp. 625–630, 1984.
[2] J. L. Willems, “A New Interpretation of the Akagi-Nabae Power Components for Nonsinusoidal Three-Phase Situations,” IEEE Transactions on Instrumentation and Measurement, vol. 41, no. 4, pp. 523–527, 1992.
[3] F. Z. Peng and J. S. Lai, “Generalized instantaneous reactive power theory for three-phase power systems,” IEEE Transactions on Instrumentation and Measurement, vol. 45, no. 1, pp. 293–297, 1996.
[4] A. Menti, T. Zacharias, and J. Milias-Argitis, “Geometric algebra: A powerful tool for representing power under nonsinusoidal conditions,” IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 54, no. 3, pp. 601–609, 2007.
[5] F. G. Montoya, R. Ba˜nos, A. Alcayde, and F. M. Arrabal-Campos, “A new approach to single-phase systems under sinusoidal and nonsinusoidal supply using geometric algebra,” Electric Power Systems Research, vol. 189, no. July, p. 106605, 2020.
[6] X. Dai, G. Liu, and R. Gretsch, “Generalized theory of instantaneous reactive quantity for multiphase power system,” IEEE Transactions on Power Delivery, vol. 19, no. 3, pp. 965–972, 2004.
[7] A. J. Ustariz-Farfan, E. A. Cano-Plata, and S. Arias-Guzman, “Identification of protection coordination break points: A power quality approach,” IEEE Industry Applications Magazine, vol. 25, no. 5, pp. 68–82, 2019.
[8] O. V. Nos, “Linear transformations in mathematical models of an induction motor by quaternions,” International Workshop and Tutorials on Electron Devices and Materials, EDM - Proceedings, vol. 1, pp. 295–298, 2012.
[9] N. Barry, “Electrical circuit analysis using four dimensional complex numbers, in the form of quaternions,” Proceedings of the Universities Power Engineering Conference, 2013.
[10] V. D. P. Brasil, A. De Leles Ferreira Filho, and J. Y. Ishihara, “Electrical three phase circuit analysis using quaternions,” Proceedings of International Conference on Harmonics and Quality of Power, ICHQP, vol. 2018-May, no. 1, pp. 1–6, 2018.
[11] O. V. Nos, A. V. Korovin, N. I. Nos, and E. S. Kucher, “Quaternion control of four-leg inverter for distribution system with harmonicproducing load,” in 2020 XI International Conference on Electrical Power Drive Systems (ICEPDS), pp. 1–6, IEEE, 2020.
[12] L. Czarnecki, “Equivalent circuits of unbalanced loads supplied with symmetrical and asymmetrical voltage and their identification,” Electrical Engineering, vol. 78, no. 3, pp. 165–168, 1995.