

# Dynamic modelling and optimal operation of the photovoltaic/thermal systems under various weather conditions

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## Abstract.

This paper aims to introduce a dynamic model for a photovoltaic/thermal system, highlighting an approach and synthesis of existing dynamic models to depict the full photovoltaic/thermal system. The model was validated using data from an experimental setup at the Institute of Energy Technology, Faculty of Energy Technology, University of Maribor, under different weather conditions. The model's accuracy was assessed through normalized Root Mean Square Error, normalized Mean Bias Error, Correlation Coefficient, and the coefficient of determination error metrics, focusing on the temperature and electrical output power of the photovoltaic/thermal modules, as well as the temperature of the thermal energy storage tank. Furthermore, the paper also focuses on the optimal operation of the photovoltaic/thermal system under different mass flow rates of the working medium.

**Key words.** the photovoltaic/thermal system, dynamic modelling, electrical and thermal energy, experimental system

## 1. Introduction

As the world grapples with various challenges ranging from energy security to environmental sustainability, there's a noticeable shift in the production of electrical and thermal energy. Driven by a diverse set of energy policies, environmental considerations, and technological advancements, there's an increasing reliance on renewable energy sources. This paradigm shift not only reflects our growing environmental consciousness but also our collective effort to reduce the carbon footprint associated with energy production. Among the myriad of renewable energy options, photovoltaic systems (PV) have seen a significant upsurge, particularly in their adoption in residential buildings. Moreover, introducing innovative solar technologies, such as photovoltaic/thermal (PV/T) systems, marks a significant milestone in the renewable energy landscape. These systems are designed to harness the sun's energy more efficiently, providing not just electrical energy but also thermal energy for residential use.

To mathematically describe the behaviour of the PV/T system, encompassing both its electrical and thermal

aspects, various methodologies can be employed. The electrical subsystem of the PV/T system can be modeled using the equivalent electrical circuit of a solar cell, as referenced in several studies [1],[2]. Meanwhile, the thermal subsystem can be represented through either static or dynamic heat transfer equations, each approach detailed in different pieces of literature [3]-[6]. While there is a substantial volume of research dedicated to the mathematical modelling of PV/T modules and Thermal Energy Storage Tanks (TEST), a comprehensive mathematical model that fully encapsulates the entire PV/T system, including its various additional components, remains elusive. The only paper that comprehensively covers the precise dynamic behaviour of a PV/T system, encompassing both PV/T modules and the TEST, is presented in [7].

In order to mathematically describe the behaviour of the PV/T system (both electrical and thermal), we can choose different approaches. The electrical subsystem of the PV/T system can be described using the equivalent electrical circuit of the solar cell [8-10], while the thermal subsystem can be described using steady-state [11]-[13] or dynamic [14]-[17] equations for heat transfer. There are already quite a lot of papers that present the mathematical modelling of the PV/T modules and thermal energy storage tanks (TEST), but none of them fully embrace the mathematical modelling of the entire PV/T systems, which includes several other components. Simonetti et al. [2] present the development and evaluation of a dynamic mathematical model for an eleven-layer PV/T module, emphasizing the accurate determination of heat transfer between the heat exchanger and the working fluid. Their model, defined in two dimensions, assumes uniform temperature distribution across each layer. This approach aligns with similar dynamic modelling efforts by Yu et al. [8] and Guarracino et al. [18], exploring temperature distribution in two-dimensional spaces. Guarracino et al. [19] further extend their research to accurately estimate the electrical and thermal energy output of PV/T modules, employing both steady-state and dynamic models. Pierrick et al. [20] introduce a model noted for its high precision, validated

under steady-state and dynamic conditions, and incorporating PV cell temperature levels and mismatch effects. Das et al. [21] propose a seven-layer dynamic model for a PV/T module that accounts for thermal contact resistance and ohmic losses within the PV cell layer. Silva et al. [22] adopt a thermodynamic modelling approach for PV/T systems through a modular strategy implemented in Matlab/Simulink, a methodology that underpins this study. Ji et al. [23] focus on a dynamic model for a PV evaporator in a solar-assisted heat pump system, detailing refrigerant conditions like pressure, temperature, and vapor quality in two-dimensional space. Lastly, Ciabattini et al. [24] introduce a dynamic model for a PV/T module, linearized and discretized for validation against both a continuous nonlinear model and empirical data from novel PV/T module measurements.

The experimental PV/T system, located at the Institute of Energy Technology, Faculty of Energy Technology, University of Maribor, features ten PV/T modules with a total installed power of 3.3 kW<sub>p</sub>. This setup includes a thermal energy storage tank of 500 litres, two heat exchangers, and a cooling unit, which is equipped for operation in winter conditions with a 35 % water-glycol mixture. The system's integrated weather station, along with various sensors and a central control unit, facilitates comprehensive monitoring and data collection every 5 minutes. The evaluation of the PV/T system's mathematical model will be conducted based on measurements from this experimental setup, focusing on its ability to accurately reflect the system's electrical and thermal performance. This streamlined approach ensures a focused assessment of the model's efficacy in representing real-world operational characteristics. Figure 1 shows the schematic representation of the connections in the experimental PV/T system, including a residential building.

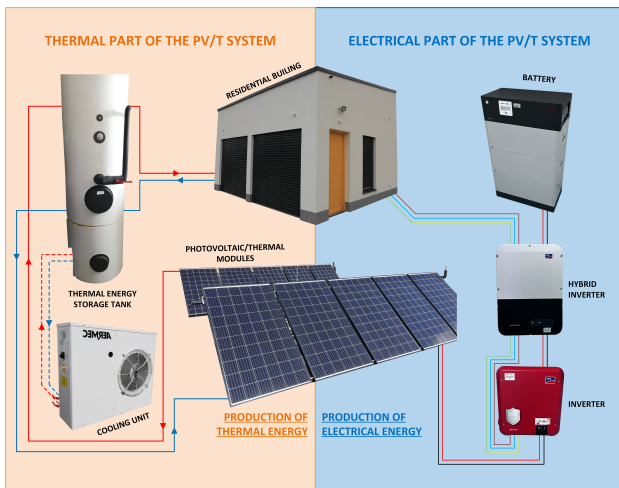


Fig. 1. Schematic representation of connections in the experimental PV/T system.

As shown in Figure 1, the PV/T system is divided into two parts: the production of electrical energy and the production of thermal energy. Other components, such as a DC/AC inverter, a hybrid DC/AC - AC/DC inverter, and a battery, are included in the electrical energy production part. An auxiliary cooling unit and TEST are included in the thermal energy production part. The building, also

shown in Figure 1, is supplied with both electrical and thermal energy by the PV/T system.

The paper is divided into four chapters, namely the introduction, methods (mathematical models for PV/T system), results (validation of the mathematical model and optimal operation of PV/T system based on the mass flow rate of the working medium), and conclusion.

## 2. Methods

The mathematical model for the PV/T system is segmented into various subsystems. It includes a mathematical model for the PV/T modules, encompassing both electrical and thermal subsystems, and a separate mathematical model for the TEST, described as a thermal subsystem. All the mentioned subsystems will be presented in more detail further below.

### A. Mathematical model of the electrical subsystem of the PV/T system

The mathematical model of the electrical subsystem of the PV/T module is described by using the equivalent circuit of a double-diode model, which is much more accurate than any other empirical equation. The output power accuracy of the PV/T module is enhanced by characterizing four electrical parameters – short-circuit current ( $I_{sc}$ ), open-circuit voltage ( $V_{oc}$ ), series resistance ( $R_s$ ), and shunt resistance ( $R_{sh}$ ) – as functions of the solar irradiance  $G$  and the temperature of the PV/T module  $T_{PV/T}$ . The double-diode model is presented in Figure 2 and by (1):

$$I = I_{ph} - I_{D1} - I_{D2} - I_{sh} \quad (1)$$

Where  $I$  represent the total current in the double-diode model,  $I_{ph}$  is the photogenerated current,  $I_{D1}$  is the current through the first diode,  $I_{D2}$  is the current through the second diode, and  $I_{sh}$  is the current through the shunt resistance.

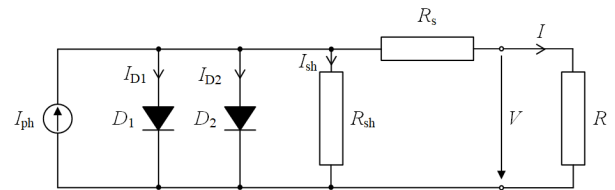


Fig. 2. Equivalent circuit of a double-diode model.

### B. Mathematical model of the thermal subsystem of the PV/T system

The mathematical model of the thermal subsystem of the PV/T module is described by using differential thermal equations over time, which is more vividly presented in our paper [7]. The input and output heat flows for each layer are precisely defined based on the heat transfer mechanism (conduction, convection, and radiation). Specified layers of the PV/T module are subjected to the entirety of the three heat transfer mechanisms (glass and protective layer – presented with (2)), while the sheet-

and-tube heat exchanger layer is exclusively influenced by convection and conduction (presented with (3)) and others (intermediate layers composed of "solid material") are governed solely by conduction (presented with (4)).

$$\left(\rho \cdot A \cdot C_p\right)_n \cdot \frac{dT_n}{dt} = Q_{\text{rad,IN}} + Q_{\text{cond}} - Q_{\text{conv,OUT}} - Q_{\text{rad,OUT,sky}} - Q_{\text{rad,OUT,ground}} \quad (2)$$

$$\left(\rho \cdot A \cdot C_p\right)_n \cdot \frac{dT_n}{dt} = Q_{\text{cond,1}} - Q_{\text{cond,2}} - Q_{\text{conv,fluid}} \quad (3)$$

$$\left(\rho \cdot A \cdot C_p\right)_n \cdot \frac{dT_n}{dt} = Q_{\text{cond,1}} - Q_{\text{cond,2}} \quad (4)$$

Where  $\rho$  represents the density,  $A$  denotes the surface area,  $C_p$  is the specific heat,  $dT_n$  indicates the temperature of the  $n$ -th layer,  $Q_{\text{rad,IN}}$  is the inlet heat flow due to radiation,  $Q_{\text{cond}}$  is the heat flow due to convection,  $Q_{\text{conv,OUT}}$  is the outlet heat flow due to convection,  $Q_{\text{rad,OUT,sky}}$  is the heat flow due to convection emitted towards the sky,  $Q_{\text{rad,OUT,ground}}$  is the heat flow due to convection emitted towards the ground and  $Q_{\text{conv,fluid}}$  represents the heat flow due to convection associated with the fluid. More detailed equations representing separate layers inside the PV/T module are presented in [7].

In the sheet-and-tube heat exchanger layer, the mass flow rate of the cooling medium, which connects from the PV/T module connection to the first heat exchanger located between the PV/T modules and the TEST, is also considered. It can be inferred that two mass flows, denoted by  $\dot{m}_1$  and  $\dot{m}_2$ , are present in the modelling of both thermal subsystems (PV/T and TEST). The cooling rate of the PV/T modules can be increased or decreased by the first mass flow  $\dot{m}_1$ , which is reflected in the production of electrical and/or thermal energy. The rate at which thermal energy is stored in the TEST can be influenced by the second mass flow, denoted as  $\dot{m}_2$ . Figure 3 presents the cross-section of the PV/T module in layers.

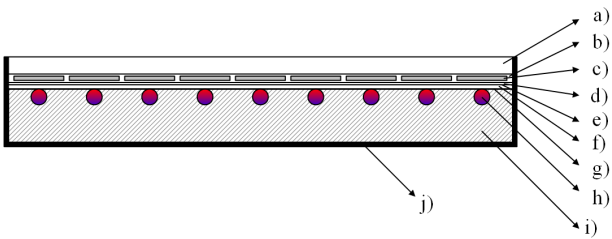


Fig. 3. Cross-section of the PV/T module in layers: a) glass, b) EVA foil, c) PV cell, d) PVF foil, e) adhesive, f) copper absorber, g) sheet-and-tube heat exchanger, h) glycol (mass flow rate  $\dot{m}_1$ ), i) glass wool, and j) protective layer. [7]

### C. Mathematical model of the thermal subsystem of the TEST

The mathematical model of the thermal subsystem of the TEST is described by using differential thermal equations over time, which is more vividly presented in our paper [7]. The input and output heat flows for each layer are

precisely defined based on the heat transfer mechanism (conduction and convection) described by (5).

$$\left(\rho \cdot v \cdot \pi \cdot r \cdot C_p\right)_n \cdot \frac{dT_n}{dt} = Q_{\text{gain}, n \rightarrow n+1} + Q_{\text{gain}, n \rightarrow n-1} - Q_{\text{loss}, n \rightarrow \text{env}} - Q_{\text{trans}, n+1} - Q_{\text{trans}, n-1} \quad (5)$$

Where  $v$  represents the height of the TEST,  $r$  is its radius,  $Q_{\text{gain}, n \rightarrow n+1}$  and  $Q_{\text{gain}, n \rightarrow n-1}$  are the input heat flows due to convection (through working medium) and conduction (through TEST wall), respectively.  $Q_{\text{loss}, n \rightarrow \text{env}}$  represents the heat flow due to convection losses from TEST to the environment.  $Q_{\text{trans}, n+1}$  and  $Q_{\text{trans}, n-1}$  denote the output heat flows attributable to convection, which corresponds to the transfer of the working medium into and out of the TEST. Figure 4 presents the cross-section of the TEST in layers.

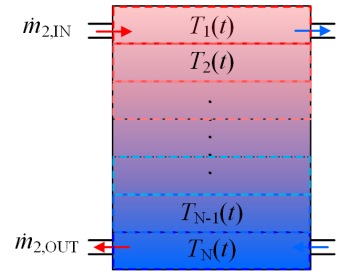


Fig. 4. Cross-section of the TEST in layers. [7]

## 3. Results

Based on the presented mathematical model of the PV/T system, the subsequent results are provided, describing the model's accuracy. As shown in Figure 5, the validation of the mathematical model of the PV/T system utilizes various quantitative metrics, such as normalized Root Mean Square Error (nRMSE), normalized Mean Bias Error (nMBE), Correlation Coefficient (CC), and the coefficient of determination ( $R^2$ ).

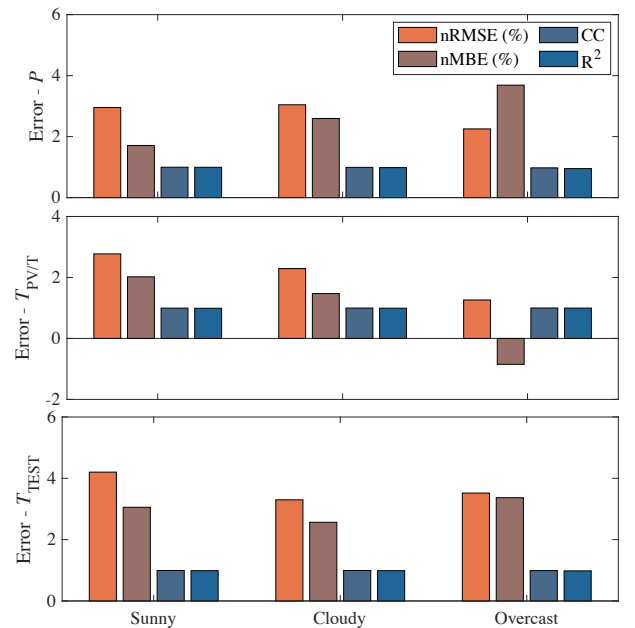


Fig. 5. Validation of the mathematical model of the PV/T system using diverse quantitative metrics.

The results from Figure 5 suggest that while the model is generally reliable and provides a strong linear correlation with actual measurements, its precision varies with weather conditions. The higher errors under sunny conditions might warrant a closer look to improve the model's robustness against varying solar irradiation levels. The model seems to perform better under less intense solar conditions (cloudy and overcast), which could be due to less thermal stress impacting the system's dynamics. Relative to other research [5],[8], the model exhibits comparable or superior precision across all three subsystems of the PV/T system's mathematical model. Figure 6 presents the comparative analysis of average daily electrical power  $P$ , the temperature of the PV/T module  $T_{PV/T}$ , and the temperature of the TEST  $T_{TEST}$  derived from the mathematical model against empirical measurements across various weather conditions: a) clear, b) cloudy, and c) overcast.

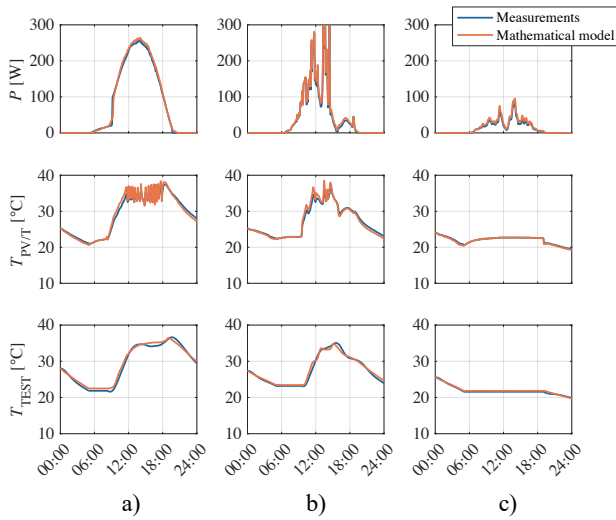


Fig. 6. Comparative analysis of average daily electrical power  $P$ , the temperature of the PV/T module  $T_{PV/T}$ , and the temperature of the TEST  $T_{TEST}$  derived from the mathematical model against empirical measurements across various weather conditions: a) clear, b) cloudy, and c) overcast.

Figure 6 substantiates the earlier commentary on the model's performance, showing a strong correlation and a good fit for both electrical power  $P$  and temperature predictions of PV/T module  $T_{PV/T}$  and TEST  $T_{TEST}$ , with some exceptions at peak values. The model demonstrates robust predictive capability across different weather conditions, especially for temperature dynamics.

Now that the mathematical model of the PV/T system has been validated with measurements, it can be used for different simulation scenarios. The main research questions are: How much electrical and thermal energy can a PV/T system produce in a certain time? And how does the mass flow rate of the working fluid in the PV/T system affect the production of electrical or thermal energy? The circulating pumps for mass flow rates  $\dot{m}_1$  and  $\dot{m}_2$  have the ability to change their mass flow rates from 0 to 0.18 kg/s. Therefore, the optimal operation of the PV/T system can be optimized using these two mass flow rates to maximize, minimize or optimize the production of electrical or thermal energy of the PV/T system, based on the consumption profile. To fully understand the behaviour of the PV/T system, the Figure 7 and Figure 8 show the daily

production of electrical  $E_{ele.}$  and thermal energy  $E_{ther.}$  of the PV/T system as a function of the change in mass flow rates of the PV/T modules  $\dot{m}_1$  and the mass flow rates of TEST  $\dot{m}_2$ .

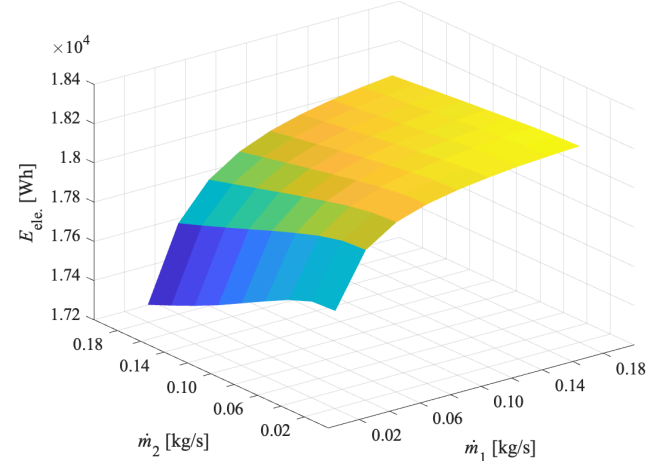


Fig. 7. Daily production of electrical energy  $E_{ele.}$  of the PV/T system as a function of the change in mass flow of the PV/T modules  $\dot{m}_1$  and the mass flow of TEST  $\dot{m}_2$ .

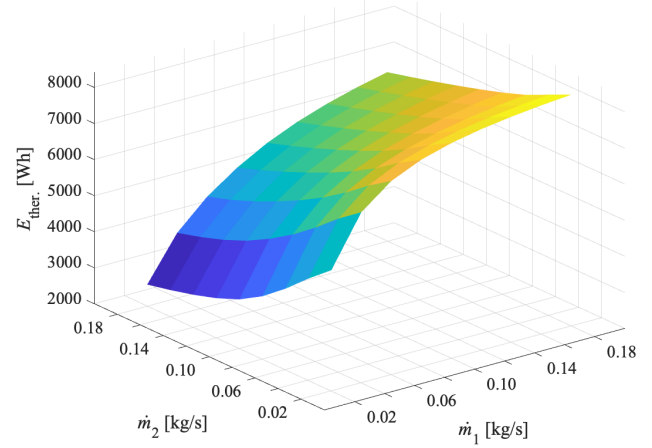


Fig. 8. Daily production of thermal energy  $E_{ther.}$  of the PV/T system as a function of the change in mass flow of the PV/T modules  $\dot{m}_1$  and the mass flow of TEST  $\dot{m}_2$ .

As shown in Figure 7, optimizing electrical energy production involves employing the highest mass flow rate  $\dot{m}_1$  on the side of the PV/T modules and maintaining the lowest mass flow rate  $\dot{m}_2$  on the TEST side. Furthermore, it's observed that the mass flow rate  $\dot{m}_2$  has a minimal impact on electrical energy generation, due to the continuous cooling effect provided by the high flow rate  $\dot{m}_1$ . Consequently, the rate at which waste heat is stored on the TEST side - whether slowly or rapidly has a negligible effect on the process.

In contrast, the production of thermal energy (shown in Figure 8) is significantly influenced by the mass flow rate  $\dot{m}_2$ . A higher mass flow rate  $\dot{m}_2$  leads to the working medium's temperature being continuously mixed with the colder working medium at the TEST's bottom, resulting in a reduction of heat generation.

Based on these results, the optimal points within the day for increasing or decreasing the production of electrical

and/or thermal energy can be determined according to the consumption profile by employing a suitable optimization algorithm.

## 4. Conclusion

This paper successfully introduces and validates a dynamic mathematical model for PV/T system, demonstrating an approach to synthesizing existing models to accurately depict the full PV/T system's behaviour. Validated with measurements under varying weather conditions, the model proves its accuracy and reliability through rigorous metrics such as nRMSE, nMBE,  $R^2$  and CC, focusing on key outputs like temperature and electrical power of the PV/T system. Moreover, the study explores the PV/T system's optimal operation, revealing that the production of electrical energy is maximally enhanced by high mass flow rates at the PV/T modules, while the production of thermal energy is significantly affected by the mass flow rate at the TEST. These insights pave the way for optimizing the system's performance based on consumption profiles. The paper contributes significantly to the field by offering a comprehensive mathematical representation of the PV/T system, thereby facilitating a deeper understanding of its operation and potential for energy efficiency and sustainability in residential applications.

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