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# Impact of Uncertainty Quantification on Small Signal Stability of Power System

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**Abstract.** The statistical uncertainty quantification of the injected uncertainties into a power system has been done to study the response of the power system to such injected randomness. The dominant electromechanical modes have been analysed and the frequency spectrum of the trajectories obtained for various state variables has been used to measure the impact of injected randomness on the system. The idea of Statistical Distance between the point of input perturbations and output measurement data has been explored. The simulation results indicate that there is a direct relationship between the input uncertainty and output measurement variable and is greatly affected by the magnitude and location of random injection. The above framework has been validated using Kundur's two area system and IEEE 14 Bus system.

**Key words.** Gaussian Mixture Models (GMM), Randomness, Sampling, Small Signal Stability, Uncertainty Quantification

## 1. Introduction

The growing shift of the energy production scenario towards a cleaner and greener future, has led to the increase in the penetration of solar and wind based generation into the current power system. This has lead to the increase in the stochastic nature of the power system owing to the random nature of wind speed and solar irradiation. In addition to this, the loads connected to the power system is also not deterministic in the entire time range leading to extra sources of randomness into the power system. The impact of such randomness can't be ignored during the system operation, planning and security assessment [1]. This leads to the whole area of Uncertainty Quantification (UQ) [2], [3] and [4].

The uncertainties in the form of either variable loads or renewable sources are not considered for Deterministic Load Flow (DLF) studies. When these uncertainties or randomness are incorporated into the analysis, the DLF studies gets modified into Probabilistic Load Flow (PLF) studies. This can be achieved either by modifying the modelling of the power system using Stochastic Differential Equations (SDEs) and Stochastic Differential Algebraic Equations (SDAEs) [5] or directly using a data driven approach. The later approach has been used in this work. Multiple trajectories or simulations needs to be run to get statistically viable results, generally using Monte Carlo simulation [6] and [7].

The present work deals with the impact of input randomness on the power system. The magnitudes of oscillations in the frequency spectrum of the desired state variables have been used to analyse the same. The frequency spectrum in the time series has been done using Fourier Transform of the data. The propagation of the input randomness on the different measurement data throughout the entire system has been found to follow a trend based on the point of injection of the noise. This trend has been validated using the idea of statistical distance. Statistical distance is used as a tool to find the statistical intimacy between input injected stochasticity and output measurement data. Lower the value of the distance, more is the impact. The above methodology has been validated on Kundur's two area system using power loads as stochastic parameter situated in different areas and IEEE 14 bus system modified to include wind based systems in order to study the statistical impact of stochastic nature of wind speed.

The rest of the paper has been organized as follows. Section II deals with the Statistical Characterization of Stochastic parameters explaining the methodology in detail. Section III comprises of the numerical results obtained by doing the Fourier analisis using Kundur's two area system. In section IV, IEEE 14 bus system has been modified to include wind-based generation and its results have been presented. Section V shows the numerical results on the modified IEEE 14 bus system including wind generation for statistical uncertainty quantification using the idea of statistical distance. Section VI finally concludes the work.

## 2. Statistical Characteristics of Stochastic Parameters

### *A. Gaussian Mixture Models*

Previous literatures have used Ornstein-Uhlenbeck Process (OUP) [6] and Fokker-Planck equation [7] for dealing with power systems subjected to random injections. Such methods deal with modifying the existing Differential Algebraic Equations (DAEs) and Differential Equations (DEs) to include the stochasticity, leading to the modelling of the power system using Stochastic Differential Algebraic Equations (SDAEs) and Stochastic Differential Equations (SDEs). However, the idea of statistical uncertainty quantification using GMMs leads to a more data driven approach, where the system modelling remains untouched. Only the statistical relationship between the input injected randomness and output measurement data is analysed. The stochasticity present in the input random injections of the power system gets manifested in the output measurement variables as well, when the sampling is done at an optimum rate [10].

Gaussian Mixture Models (GMMs) have been used for modelling the input injected randomness into the system and study its statistical properties which will lead to the statistical uncertainty quantification of the entire system. These models can be used to fit complex uncertainties related data which can't be fit using any single standard probability distribution function. This versatile property of the GMMs makes them superior than single uni-modal normal distribution function [8] and [9]. A GMM represents the randomness in the form of a white noise which follows a gaussian or normal distribution. Each normal distribution within the GMM is considered to be its components which is defined individually by its weight or mixing proportion and mean. The number of components which will comprise the entire GMM is decided by Akaike's Information Criteria (AIC) [11].

### *B. Statistical Distance*

Statistical Distance gives the measure of how strongly the output measurement data would be affected by the randomness of the input stochastic variable. It can be used to measure the statistical intimacy between two statistical objects or variables. The larger the statistical distance between the input randomness and output measurement data, the lower will be the impact of the randomness on the output parameter.

## 3. Numerical Results on Kundur's Two Area System

Kundur's two area system has been used to carry out the analysis. The total simulation time is 200 seconds. All the simulations were carried out using Power System Analysis Toolbox (PSAT) [12] and Simulink in MATLAB. GMM based random load uncertainties were injected into the system. A random Gaussian signal with zero mean has been used to add noise into the load power in order to make it stochastic in nature. Details of Statistical Characterization using GMMs has already been discussed in the previous section. Gaussian noise has been added at the load buses. The loads connected at Bus 7 and 9 has been considered as the stochastic variables.

Four cases of stochastic processes with varying Signal to Noise Ratio (SNR) has been considered as follows:

Case 1: Stochastic Load at Bus7, 20 SNR.
Case 2: Stochastic Load at Bus9, 30 SNR.
Case 3: Stochastic Load at Bus7, 30 SNR.
Case 4: Stochastic Load at Bus9, 35 SNR.

### *A. Stochastic Load at Bus 7:*

The dominant electromechanical modes of the system upon introduction of noise at Bus 7 are shown in Table I and II for cases 1 and 3 respectively. The modes are obtained with the addition of stochastic real load power injections at Bus 7 through GMM based modeling. The active power injections *Pg* of synchronous generator of G1 (area 1) and G4 (area 2) for cases 1 and 3 are shown in Fig. 1 and Fig. 3 respectively.

The magnitude of the oscillations in the frequency spectrum of the synchronous machines also depends on their participation factors. For modes in Tables I and II, the participation of G1 is higher as compared to G4 as the point of noise injection is in Area 1.

TABLE I
DOMINANT ELECTRO-MECHANICAL MODES OF TWO AREA SYSTEM WITH STOCHASTIC LOAD IN AREA 1 CASE 1

| Mode | Freq (Hz) | Damping ratio (%) |
|---|---|---|
| 1 | 0.567 | -7.905 |
| 2 | 1.0148 | -18.27 |
| 3 | 1.031 | -15.828 |

TABLE II
DOMINANT ELECTRO-MECHANICAL MODES OF TWO AREA SYSTEM WITH STOCHASTIC LOAD IN AREA 1 CASE 3

| Mode | Freq (Hz) | Damping ratio (%) |
|---|---|---|
| 1 | 0.572 | -7.09 |
| 2 | 1.006 | -16.331 |
| 3 | 1.029 | -14.354 |

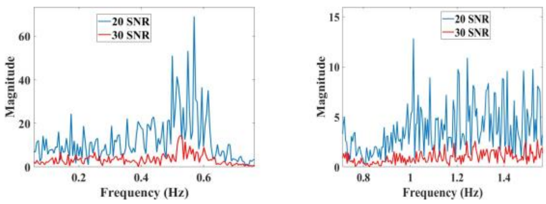


Fig. 1: Frequency spectrum of *Pg* of synchronous generator G1 for cases 1 and 3

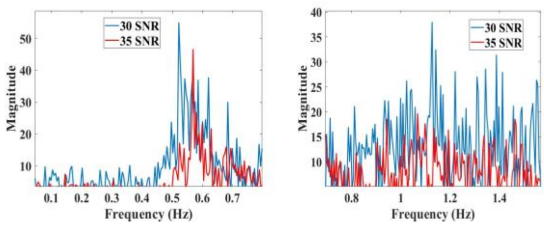


Fig. 2: Frequency spectrum of *Pg* of synchronous generator G1 for cases 2 and 4

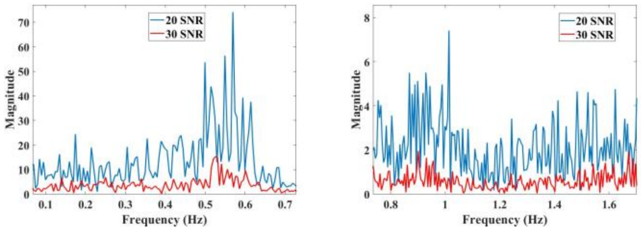


Fig. 3. Frequency spectrum of *Pg* of synchronous generator G4 for Cases 1and 3

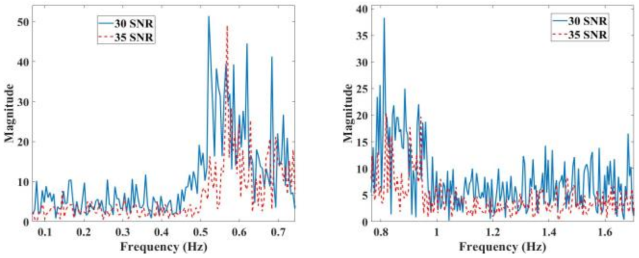


Fig. 4. Frequency spectrum of *Pg* of synchronous generator G4 for Cases 2and 4

*B. Stochastic Load at Bus 9:*

The dominant electromechanical modes of the system upon introduction of noise at Bus 9 are shown in Table III and IV for cases 2 and 4 respectively. The active power injections *Pg* of synchronous generator of G1 (area 1) and G4 (area 2) for cases 2 and 4 are shown in Fig. 2 and 4 respectively. For modes in Tables III and IV, the participation of G4 is higher as compared to G1 as the point of noise injection is in Area 2.

It is quite evident from the above figures, that the amplitude of the oscillations of the generators increases on decreasing the value of SNR of the additive white gaussian noise in the load power. It was found that the combined participation factors of G1 and G2 is greater than that of the combined participation of G3 and G4 for cases 1 and 3. The reverse happens for cases 2 and 4 when the point of stochastic load injection is at Bus 9 in Area 2.

TABLE III
DOMINANT ELECTRO-MECHANICAL MODES OF TWO AREA SYSTEM WITH STOCHASTIC LOAD IN AREA-2 CASE 2

| Mode | Freq (Hz) | Damping ratio (%) |
|---|---|---|
| 1 | 0.571 | -8.029 |
| 2 | 1.0229 | -16.961 |
| 3 | 1.04 | -18.622 |

TABLE IV
DOMINANT ELECTRO-MECHANICAL MODES OF TWO-AREA SYSTEM WITH STOCHASTIC LOAD IN AREA 2 CASE 4

| Mode | Freq (Hz) | Damping ratio (%) |
|---|---|---|
| 1 | 0.557 | -6.194 |
| 2 | 1.0034 | -16.054 |
| 3 | 1.0192 | -13.786 |

## 4. Numerical Results of IEEE-14 Bus system with DFIG

IEEE 14 Bus system with five synchronous machines connected at Buses 1, 2, 3, 6 and 8 respectively has been used here for studying the impact of variable wind speed using Doubly Fed Induction Generator (DFIG). Here the variable wind speed has been considered as the random variable. Weibull distribution with a scale factor of 20 and shape factor of 2 has been used to model the wind speed with a mean speed of 15 m/s. Figure 6 gives the wind speed over the span of time. The system has been implemented using PSAT/MATLAB integrated software environment. The entire simulation has been run for 100 seconds. Three scenarios have been analysed where the synchronous generators have been replaced by DFIGs connected at Bus 1, 3 and 6 respectively, one at a time. The modified IEEE 14 Bus system where the synchronous generator at Bus 3 has been replaced by a DFIG model (wind farm consisting of 300 wind turbines) of same capacity has been displayed in figure 5. Here the DFIG has been connected to Bus 3 using a new bus 15 via a transformer with unity tap ratio.

The dominant electromechanical modes of the system upon introduction of variable wind speed for the three above mentioned scenarios are shown in Tables V, VI and VII respectively. The active power *Pg* of the synchronous generators for all the scenario 1 and 2 are shown in figures 7 to 10. It can be noticed that the magnitude of the oscillations in the frequency spectrum of the synchronous machines varies as the point of placement of DFIG changes. The machines which are closer to the DFIGs show a greater amplitude of oscillations as compared to the ones farther away from it. As a result it can be concluded that the point of injection of randomness into a system plays a vital role in the propagation of the noise associated with it.

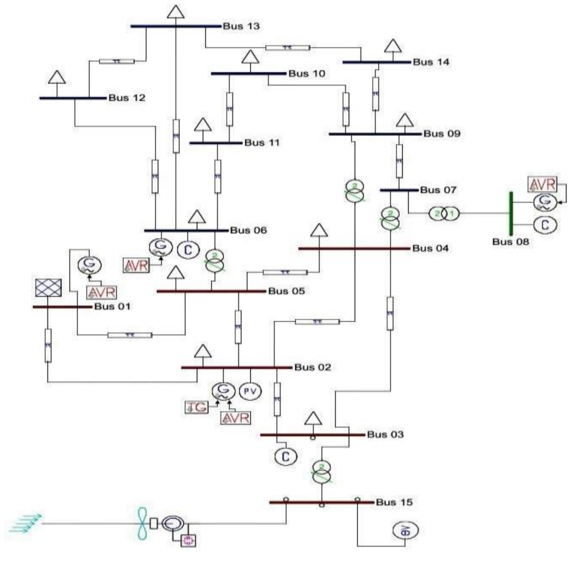


Fig. 5. Modified IEEE 14 Bus system with DFIG at Bus 3

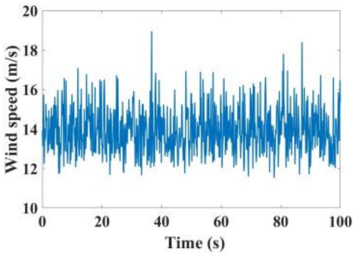


Fig. 6. Wind speed for DFIG

TABLE V
DOMINANT ELECTRO-MECHANICAL MODES OF THE MODIFIED IEEE 14 BUS SYSTEM WITH DFIG CONNECTED AT BUS 1

| Mode | Freq (Hz) | Damping ratio (%) |
|---|---|---|
| 1 | 0.231 | -53.949 |
| 2 | 0.508 | -21.526 |
| 3 | 0.762 | -12.609 |
| 4 | 1.727 | -8.239 |

TABLE VI
DOMINANT ELECTRO-MECHANICAL MODES OF THE MODIFIED IEEE 14 BUS SYSTEM WITH DFIG CONNECTED AT BUS 3

| Mode | Freq (Hz) | Damping ratio (%) |
|---|---|---|
| 1 | 0.364 | -45.498 |
| 2 | 0.419 | -14.818 |
| 3 | 1.6073 | -26.385 |
| 4 | 1.687 | -0.845 |

TABLE VII
DOMINANT ELECTRO-MECHANICAL MODES OF THE MODIFIED IEEE 14 BUS SYSTEM WITH DFIG CONNECTED AT BUS 6

| Mode | Freq (Hz) | Damping ratio (%) |
|---|---|---|
| 1 | 0.337 | -51.361 |
| 2 | 0.351 | -17.912 |
| 3 | 1.238 | -29.315 |
| 4 | 1.635 | -30.455 |

## 5. Numerical Results for Statistical Uncertainty Quantification

The statistical distance between input random wind speed and output power of the synchronous machine for scenario 1 with DFIG at bus 1 and scenario 2 with DFIG at bus 3 are shown in Tables VIII and IX. The statistical distance has also been shown using different combination of output parameter of bus voltage and input wind speed for scenario 2 in Table X.

Lower value of statistical distance signifies more intimate behaviour between input and output parameters and vice versa. If the statistical distance reduces, the impact of randomness on the output parameter increases, leading to greater amplitude of modes in the Fourier analysis. As a result, the amplitude of the modes in the frequency spectrum analysis shown in section IV should be inversely proportional to the statistical distance analysis. This has almost been achieved using the analysis presented here. For example, the amplitudes of the modes of generators at Buses 1 and 2 are greater than the generators at Buses 6 and 8 in figures 9 and 10 respectively. Similar trends can also be seen in the statistical distance in Table X, where the statistical distance for buses 1 and 2 are lesser than that of the buses 6 and 8 for the same scenario.

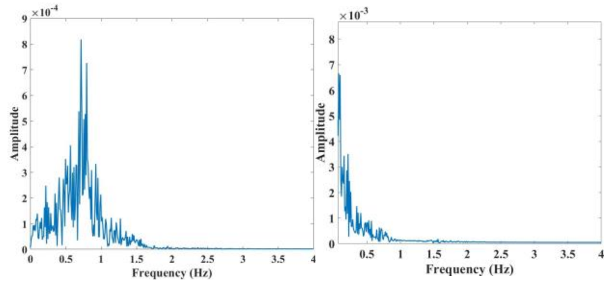


Fig. 7. Frequency spectrum of *Pg* of synchronous generators at bus 3 and 2 for DFIG at Bus 1

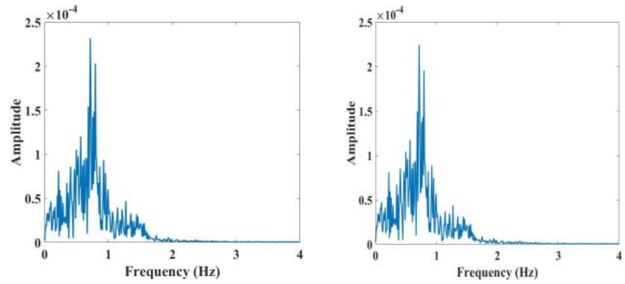


Fig. 8. Frequency spectrum of *Pg* of synchronous generator at bus 8 and 6 for DFIG at Bus 1

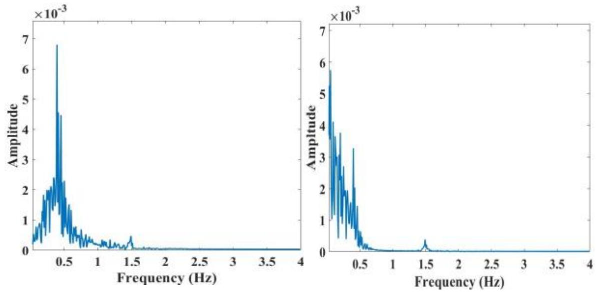


Fig. 9. Frequency spectrum of *Pg* of synchronous generator at bus 1 and 2 for DFIG at Bus 3

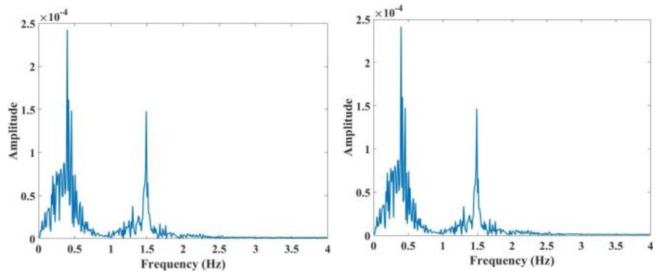


Fig. 10. Frequency spectrum of *Pg* of synchronous generator at bus 8 and 6 for DFIG at Bus 3

TABLE VIII
STATISTICAL DISTANCE FOR DFIG AT BUS 1

| Cases | Distance |
|---|---|
| Speed & Generator Power at Bus 2 | 0.528 |
| Speed & Generator Power at Bus 3 | 0.92751 |
| Speed & Generator Power at Bus 8 | 0.92752 |
| Speed & Generator Power at Bus 6 | 0.92752 |

TABLE IX
STATISTICAL DISTANCE
FOR DFIG AT BUS 3

| Cases | Distance |
|---|---|
| Speed & Generator Power at Bus 2 | 0.0408 |
| Speed & Generator Power at Bus 8 | 0.4375 |
| Speed & Generator Power at Bus 6 | 0.4375 |
| Speed & Generator Power at Bus 1 | 2.0301 |

TABLE X
STATISTICAL DISTANCE
FOR DFIG AT BUS 3

| Cases | Distance |
|---|---|
| Speed and Voltage at Bus 3 | 0.5742 |
| Speed and Voltage at Bus 2 | 0.6092 |
| Speed and Voltage at Bus 1 | 0.6242 |
| Speed and Voltage at Bus 6 | 0.6342 |
| Speed and Voltage at Bus 8 | 0.6342 |

## 6. Conclusion

The present work studies the impact of randomness of the stochastic process on the dynamic response of the power system. The frequency spectrum obtained using Fourier Transform for the various state variables of the power system has been used to carry out the analysis. The stochastic nature of the random variable has been modelled using GMMs and the concept of statistical distance has been used to quantify it. Kundur's two area system has been used to show that the presence of stochastic variable in an area of the system has a reduced impact on the modes of the other areas as has been evident from the magnitude of the modes as well as the participation factors of the individual synchronous machines. The magnitude of the modes of the oscillations is also a function of the amount of randomness injected. The lesser the SNR, the higher was the magnitude of oscillations owing to the increased amount of noise in the stochastic variable.

The modified IEEE 14 Bus system was used to model the variable nature of the wind speed in case of renewable integrated power systems. The placement of the DFIG plays a vital role as it changes the point of injection of randomness into the system. This has a direct impact on the magnitude of the oscillations and varies with the point of randomness. The idea of statistical distance between the input variable wind speed and output power generation of the synchronous machines or the output bus voltages have been used as a tool to find the statistical intimacy. Lower values of statistical distance signifies more statistical dependency amongst the input random variable and output measurement data. The results of the Fourier analysis and statistical analysis using statistical distance leads to similar trends of results.

Such analysis gives a direct data driven one on one relationship between the input injected randomness and output measurement data in the power system. It is helpful in analysing the propagation of a noise and can lead to a better understanding of its nature leading to better control strategies for the overall system. The current methodology can be used to model any kind of uncertainty injected into the power system including cases with variable irradiation in case of solar photovoltaic integrations.

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