20th International Conference on Renewable Energies and Power Quality (ICREPQ'22)
Vigo (Spain), 27th to 29th July 2022
Renewable Energy and Power Quality Journal (RE&PQJ)
ISSN 2172-038 X, Volume No.20, September 2022

Multi-objective optimisation of technical wind turbines parameters based on multi-physical models

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Abstract. This paper presents a decision tool for the optimisation of wind turbine technical parameters, using a multi-physical model of the power system. This includes a multi-physical modelling of aerodynamical, mechanical and electro-mechanical system behaviours. The aerodynamics is based on a blade element momentum model. A mass model of wind turbines components is also used in this paper. The optimisation is performed with NSGA-II algorithm which may choose technical parameters (blade length, transmission ratio or electro-mechanical coefficient in this example) to maximise performances indicators (in this example the output electrical power and the wind turbine mass). The results provide a wide range of solutions that are the best compromises between the performances indicators chosen. The diversity in terms of parameters allows great latitude in the design of wind turbine.

Key words. Wind-turbine, multi-physical modelling, multi-objective optimisation, renewable energy system, genetic algorithm

1. Introduction

In the current energy production challenging context, Wind Turbines (WT) appear to be an interesting option since this technology provides energy from a renewable source with a very low emissions factor over his life cycle [1]. The modelling of WT differs widely depending on the technical domain highlighted by the study. First some papers model the aerodynamical part [2],[3]. Others authors model aerodynamical behaviours with simplified analytic expressions and develops more widely the generator part [4],[5]. Concerning the aerodynamical section, there are different ways (like blade element momentum, vortex model or computational fluid dynamics) to model the wind interaction with the blades, which implies different computational costs [6].

Chehouri *et al.* [7] list different methods of optimising wind turbines. In the literature, different kinds of optimisations are performed with diverse objectives. Most of optimisations minimise the cost of energy (COE) of the WT [8],[9]. Some others authors propose to maximise the power obtain with a peculiar wind speed [10] or the anual energy production (AEP) depending on a wind distribution [11]. Besides, some authors focused on the optimisation of more technical objectives such as the blade mass [12]. Finally, several authors focuses on multi-objective modelling with objectives as AEP maximisation, mass minimisation or COE minimisation [13],[14].

Optimisation algorithm are a wide family of algorithms that tries to minimise an objective function [15]. In our case, we do not have explicit derivable function, so we cannot use gradient-based method. We have to use metaheuristic algorithms. In the metaheuristic algorithms group, we can find main families of algorithms such as Particle Swarm Optimisation (PSO), Ant Colony Optimisation (ACO) and Genetic Algorithms (GA). Among Genetic algorithms, algorithms can focus on one or many objectives. The latter are called multi-objective genetic algorithms (MOGA). Fast-elitist Non dominated-sorted algorithm (NSGA-II) is a popular and reliable algorithm able to perform a multi-objective optimisation [16],[17].

This work provides a tool able to propose technical solutions (coupling multi-physical parameters) in order to optimise performance indicators, *e.g.* a high output power and a low system mass. We first propose a multi-physical model of a wind turbine coupling electro-mechanical models with a blade element momentum (BEM) theory for the aerodynamic model. We also propose a mass model of the WT components. This section aims to present the objectives functions used in the optimisation algorithm. The next section implement the models in a multi-objective optimisation algorithm (NSGA-II) in order to improve the power output of the system while minimising the overall mass. Finally, a practical example is presented in the last section, which highlights the advantages of the tool.

2. Multi-physical models

A. Assumptions for the models

This paper proposes a multi-physical model composed of aerodynamical, mechanical and electro-mechanical parts. A mass model is also used. The following models of this paper are constructed under some assumptions. For the

aerodynamical part, commonly used assumptions are the consideration of a steady state, an incompressible flow and a perfect fluid. The upcoming wind (V_0) is considered unidirectional and uniform over the rotor surface (S). For mechanical part, we use a perfect model of gearbox with no power losses in a steady state. The electro-mechanical part describe an ideal generator without losses and in a steady state. The electrical load is considered as a resistive load.
From the perspective of our optimisation in the section n°4, we assume that static power proportional losses attributable to mechanical and electro-mechanical systems lead to a uniform downsizing performances over WT population. This drives to the assumption that losses do not affect the repartition of best individuals over the whole WT population in the optimisation process.

B. *Aerodynamical model*

Among aerodynamical models, BEM combine both momentum and blade elements methods. The model used come from the work of Hansen [18] and Liu [19]. Global parameters of the WT are represented such as the number of blades (B) or propeller radius (R_{prop}). Moreover, blades are discretized in several blade elements. Figure 1 shows one of this element with a length *dr*, a local cord (c) and at a distance *r* from the centre of the WT (O). $\overrightarrow{\boldsymbol{e_r}}$ and $\overrightarrow{\boldsymbol{e_\theta}}$ represent respectively the radial and the tangential direction (in the rotational plan).
Figure 2 presents the relative wind speed (V_r) seen by the profile at the local radius (r). V_r is composed of two speeds V_a and V_t (Eq.1). V_a is the axial velocity at blade's position. V_t is the tangential velocity seen by the element's blade because of its rotation (ω_{prop}). The angle between relative wind speed and the cord is called the angle of attack (α) whereas the angle between the cord line and the rotational plan is the pitch angle (β).

$$V_r = \sqrt{V_a^2 + V_t^2} \tag{1}$$

V_a and V_t can be defined with aerodynamical induction coefficients *a* and *a'* respectively axial and tangential coefficients (Eq. (2) and (3)). These coefficients cannot be found in a direct method as they result of an equilibrium between the behaviour of the flow and the presence of the WT. That is why we will use an iteration method to find them.

$$V_a = (1 - a).V_0 \tag{2}$$
$$V_t = (1 + a').\omega_{prop}.r \tag{3}$$

The power contained in the upstream wind through a WT sweeping a surface S is defined in Eq. 4.

$$P_{wind} = \frac{1}{2}\rho.S.V_0^3 \tag{4}$$

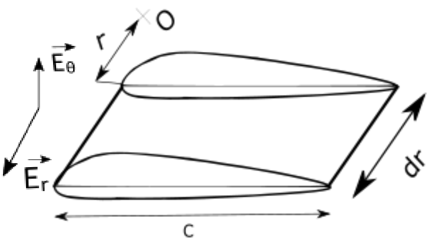

Figure 1: Blade element

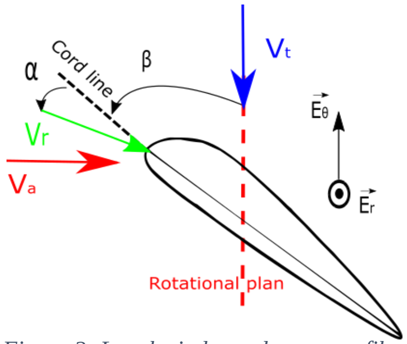

Figure 2: Local wind speeds on a profile

Important part of the BEM method is how to find the induction coefficients mentioned above. In our case, they are found through a fixed-point algorithm that iterates different values of *a* and *a'* until the convergence is ensured for each element. This convergence method provides good results and is convenient to implement although some convergence errors can occur [20]. Is such cases, we assume that no induction occurs for the elements involved. For this algorithm we first initialise guess values for *a* and *a'* (0 in our case). Then we calculate the total angle between the relative airflow and the rotational plan (Φ). Knowing the pitch angle as an input data, we can calculate the attack angle between the blade and the relative wind. Local aerodynamical coefficients C_l and C_d respectively for lift and drag are read in aerodynamical tables using the attack angle, the local Reynolds number, and the local profile. Then we can calculate the new *a* and *a'* coefficients given by these C_l and C_d numbers with equations (5) and (6). In these equations, n represents the current iteration number during the convergence procedure. The difference with previous induction coefficients is also calculated. The convergence is checked through a converge criterion (ε). The algorithm is summarised in Figure 3.
The aerodynamical tables are provided by the Heliciel software database.

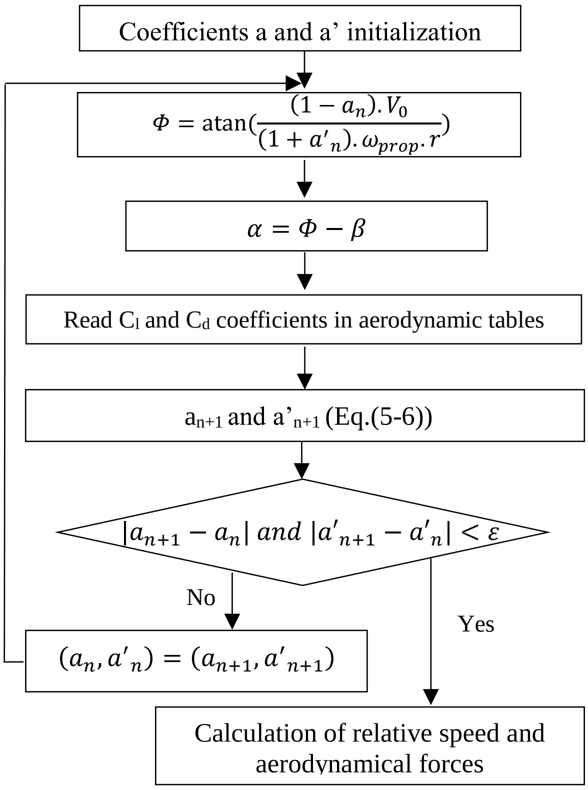

Figure 3: Fixed-point algorithm

$$a_{n+1} = \frac{1}{\frac{4.\sin^2(\Phi)}{s.[C_l.\cos(\Phi) + C_d.\sin(\Phi)]} + 1} \quad (5)$$

$$a'_{n+1} = \frac{1}{\frac{4.\sin(\Phi).\cos(\Phi)}{s.[C_l.\sin(\Phi) - C_d.\cos(\Phi)]} - 1} \quad (6)$$

With s the local rotor solidity defined as the ratio of total blade cords at this local radius over the perimeter drawn by the local radius (Eq.7).

$$s = \frac{c.B}{2.\pi.r} \quad (7)$$

The aerodynamical coefficients allows calculating the lift and drag forces that depends on the local relative wind flow. These forces need to be projected to find forces applied from the flow to the blade's element along tangential (F_t) and normal (F_a) directions (Eq.(8-9)).

$$F_t = \frac{1}{2}.\rho.c.dr.V_r^2.[C_l.\sin(\Phi) - C_d.\cos(\Phi)] \quad (8)$$

$$F_a = \frac{1}{2}.\rho.c.dr.V_r^2.[C_l.\cos(\Phi) + C_d.\sin(\Phi)] \quad (9)$$

With these forces and the local radius, we have the torque developed by every blade element. Sum of these torques (C_{wti}, for the i[th] element) along the blades and the current rotational speed provides the power developed by the turbine (Eq.10):

$$P_{prop} = \omega_{prop}.\sum_i C_{wti} \quad (10)$$

The power coefficient (C_p) of the WT is defined as a ratio between upstream wind power and mechanical power extracted by the blades (Eq.11).

$$C_p = \frac{P_{wind}}{P_{prop}} \quad (11)$$

Some corrections of the BEM model are taken into account. For high loads, *i.e.* for axial induction factors above critical value of 0.2 (a_c), the determination of the (n+1)[th] axial factor is corrected according Spera's correction (Eq. 12) [21].

$$a_{n+1} = \frac{1}{2}[2 + A(1 - 2a_c) - \sqrt{(A(1 - 2a_c) + 2)^2 + 4(A.a_c^2 - 1)}] \quad (12)$$

With

$$A = \frac{4.\sin^2(\Phi)}{s.[C_l.\cos(\Phi) + C_d.\sin(\Phi)]} \quad (13)$$

Besides, to take into account the finite number of blades and the lift induced drag at the blade's tip, a loss factor is implemented at each local radius (r) called Prandtl's tip loss factor as described in Eq. 14 ([18],[19]).

$$F = \frac{2}{\pi}\arccos(e^{-f}) \quad (14)$$

With

$$f = \frac{B}{2}.\frac{(R_{prop} - r)}{r.\sin(\Phi)} \quad (15)$$

This coefficient reduces aerodynamics performances of the few elements closest to the blade's tip.

BEM theory provides reliable results and its low computational complexity [6] allows performing it many times in metaheuristic methods.

C. Mechanical and electro-mechanical models:

The mechanical transmission is modelled by a gear ratio (red) that links propeller rotational speed ω_{prop} and generator rotational speed ω_{gen} (Eq.16) as the respective torques C_{prop} and C_{gen} (Eq.17).

$$\omega_{gen} = red.\omega_{prop} \quad (16)$$

$$C_{gen} = \frac{C_{prop}}{red} \quad (17)$$

The electro-mechanical conversion is modelled by a couple (k,k') of electro-mechanical coupling coefficients. The electromotive force of the generator E and the electro-mechanical torque C_{em} are defined in Eq. 18 and 19.

$$E = k.\omega_{gen} \quad (18)$$

$$C_{em} = k'.I \quad (19)$$

The voltage generator dissipates energy in a resistive electro-mechanical load (R_L) with a current I flowing through it and a voltage U_L across the resistance as described by Eq.20.

$$U_L = R_L.I = E \quad (20)$$

The stationary state assumption provides the generator and electro-mechanical torques equilibrium (Eq. 21).

$$J_\Delta.\frac{d\omega_{gen}}{dt} = C_{gen} - C_{em} = 0 \quad (21)$$

With J_Δ the moment of inertia at the generator rotor. Equation 21 is the equation to check to find the operating point of the system and its corresponding rotation speed $\boldsymbol{\omega_{prop}}$. This leads to Eq.22 coming from the system made by equations (11) and (16) to (21):

$$C_p = K.\omega_{prop}{}^2 \quad (22)$$

With

$$K = \frac{k.k'.red^2}{R_L.P_{wind}} \quad (23)$$

If Eq. 22 is verified by different values of ω_{prop}, we choose the highest one because it is the one that ensure the greatest C_p value. Once the current C_p corresponding at the particular functioning point has been found, the output WT power is then calculated (Eq.11) that leads to the electrical power dissipated in the resistive load generated (Eq. 24).

$$P_L = I.U_L \quad (24)$$

D. Mass model

We model the mass of the WT with analytical equations that link the mass of elements with the nominal power output of the wind-turbine (P_{nom}) (Eq. 25), the blade radius (R_{prop}), the total diameter (D_{prop}) and the low-speed shaft torque (C_{wttot}). The total mass is the sum of the masses of the components listed below :

$$P_{nom} = E.I \tag{25}$$

- Blades :

$$M_B = B.0.1452 \times R_{prop}{}^{2.9158} \tag{26}$$

- Hub :

$$M_H = 0.954 \times \frac{M_B}{B} + 5680.3 \tag{27}$$

- Low-speed shaft :

$$M_{LSS} = 0.0284 \times D_{prop}{}^{2.888} \tag{28}$$

- Gearbox :

$$M_{Gb} = 70.94 \times C_{wttot}{}^{0.759} \tag{29}$$

- Generator

$$M_{Gen} = 6.47 \times P_{nom} \tag{30}$$

M_B, M_H, M_{LSS}, M_{Gb} and M_{Gen} are respectively the masses for the total number of blades, the hub, the low speed shaft, the gearbox and the generator. For the blades, the gearbox and the generator, different mass models are available. In those cases, we took respectively the baseline model for the blades, a three-stage Planetary/Helical model for the gearbox and a three-stage with high-speed generator. Equations of mass can be found in the NREL scaling cost and models [22].

3. Multi-objective optimisation methods

The multi-objective optimisation algorithm used is NSGA-II [16]. It is based on the genetic evolution of species. The algorithm allows exploring the whole design space and has a good convergence. It presents the advantage to non-weighting the antagonist objectives: the results are compromises between them. This algorithm make evolve individuals represented by their genes (in this example an individual is a WT defined by his genes that are technical parameters) over a few generations in order to find a set of different WTs that are compromises between performance indicators objectives (in this example a high output power and a low WT mass). It is important to highlights that the optimisation algorithm can run with an infinite possibility of objective functions. The models described in section 2 are implemented as two specific objective functions.

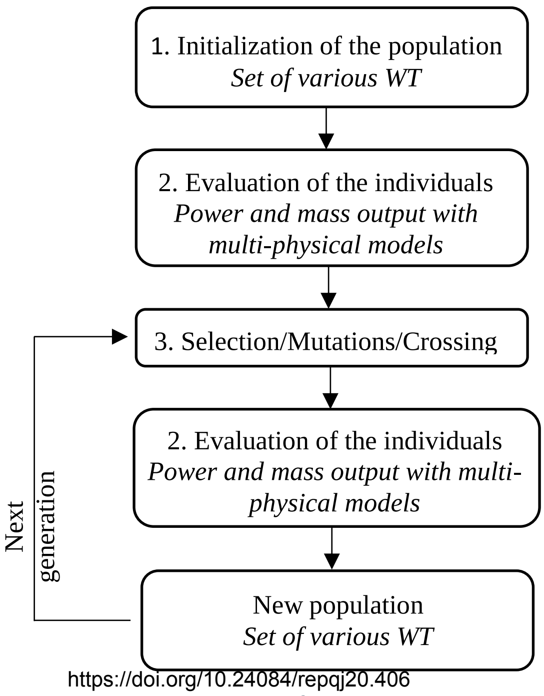

Figure 4: Functioning of the genetic algorithm NSGA-II

NSGA-II randomly uses genetic operators such as crossing and mutations to make the population evolve. The results are taken from the last generation, which gives the best WTs. They can be arranged along a so-called Pareto front. The operation of this algorithm can be describe according to Figure 4. The algorithm end when the number of generation previously set is reached.

4. Results and discussion

A. Case study

Many technical parameters could be set in the model. The complete list of them is detailled below :

- The number of blades
- The length of blade
- The length of the hub
- The pitch angle at each element
- The cord distribution along the blade
- Profile at each element
- Gear ratio
- Electro-mechanical coupling coefficients k and k'
- Resistance load

The optimisation objectives are the following: maximise the output power and minimise the total mass of the system. These objectives are antagonists, which will more likely lead to compromises. A low mass for the components chosen is considered to be a great advantage concerning the dimensioning of the whole WT structure (tower and foundations), for reducing the use of resources and the price of the system as well. The output power is calculated in a stationary state.

The optimisation parameters chosen are from different technical parts of the WT, which is made possible with the multi-physical model used:

- Propeller radius ∈ [40;80] (m)
- Gear ratio ∈ [50;200]
- k coupling coefficient ∈ [3;6] (V.s)

The NSGA-II algorithm can take continus values over these ranges. Nevertheless, to ensure that technically different solutions are tested (*i.d.* not too close), we round parameter with the tolerance value associated : respectively to the nearest unite for R_{prop}, the same for *red* and to the hundredth for *k*.

The number of elements of the blade is set to 10 and the upstream wind speed is set to 10 m/s. The profil is uniformly choosen as a NACA 0006 along the blade. Pitch angles are fixed for each elements between 0.48 rad for the most twisted and 0.018 rad for the less twisted one.
The optimisation parameters are set to 100 individuals over 300 generations which ensure the convergence of the algorithm [23].

B. Results

Figure 5 presents the results arranged along a Pareto front: each point of the front is a WT, defined by its output power in ordinate and its mass in abscissa.

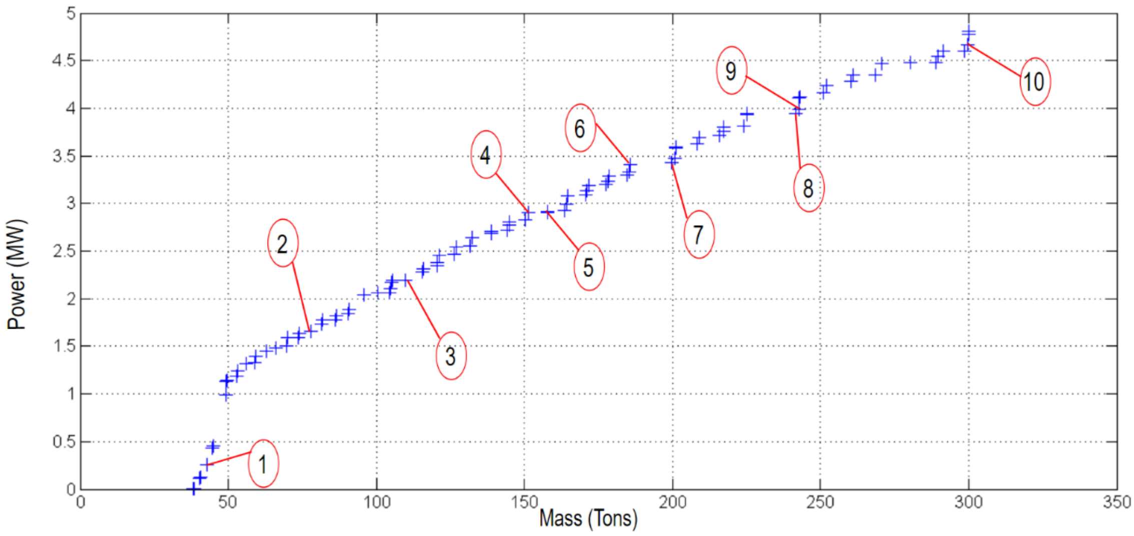

Figure 5: NSGA-II optimisation results

The whole front displays the WT population at the end of the optimisation process, which means they are the best compromises between the objectives functions. On the bottom left corner, we can see low power lightweight WT while the top right corner gathered the more powerful and massive WT. The numbers circled in red are some WT identified in the results graph and whose characteristics are listed in tables 1 and 2. Table I shows these WT represented by their parameters whereas Table II show the same WT with their performances according the objectives functions. These WT are selected arbitrary in order to show the variety of results and to simplify the analysis.

The results show a great variety of WT (see Table I). The smallest WT has a radius of 40m with a gear ratio of 51 and an electro-mechanical coefficient of 3.32 V.s. The tallest WT has twice the radius with 80m for a gear ratio of 176 and a coupling coefficient of 4.88 V.s.

The results show a wide output power and WT mass as well (see Table II). The lowest power is 0.258 MW for a mass of 42.64 tons while the biggest is about 300 tons for a power of 4.88 MW. In the Pareto front, every point is a compromise, which means that every WT dominates every other with at least one objective. The results show that depending on the compromises we want to take between mass and power, we have a full range of WT available.

Table I: WT and their parameters

WT n°	Radius (m)	Gear ratio	k (V.s)
1	40	51	3.32
2	48	68	4.61
3	55	86	4.54
4	62	110	4.56
5	63	142	3.76
6	67	146	3.89
7	69	138	4.48
8	74	155	4.65
9	74	154	4.79
10	80	176	4.88

Table II: WT and their performances

WT n°	Mass (Tons)	Power (MW)
1	42.64	0.258
2	77.85	1.66
3	109.8	2.20
4	151.4	2.90
5	157.9	2.91
6	185.6	3.41
7	199.9	3.43
8	241.6	3.95
9	242.5	3.99
10	299.7	4.88

Tables II also reveals that we can found close power output WT with a difference in term of mass. For example WT n°6 and n°7 for which power output is very close (3.41 MW for WT n°6 against 3.43 MW for WT n°7) while the difference of mass is up to 15 tons (185.6 tons for WT n°6 and 199.9 tons for WT n°7). This shows the advantage of a non-weighted multi objective algorithm where an incorrectly set up ponderate algorithm could have miss one of these two WT.

The results also show WT with different parameters but close objective performances. See WT n°4 and n°5, which have comparable performances with power of respectively 2.90 MW and 2.91 MW and masses respectively 151.4 tons and 157.9 tons. Their gear ratio and the coupling coefficient (k) parameters are significantly different: respectively 110 and 142 for the gear ratio, respectively 4.56 V.s, and 3.76 V.s for the coupling coefficient. These differences in the coefficients may results in different technical and manufacturing solutions for of gearbox and generator. This gives great latitude for the user in the WT design.

Conclusion

This paper presents a decision tool for wind turbines system that provides a wide range of WT that are the best compromises between objective functions. It uses multi-physical model, which allows setting different physical domains parameters. The aerodynamical model is based on the BEM theory solved through a fixed-point algorithm. The optimisation algorithm used is a genetic based algorithm called NSGA-II.

The results of the optimisation process provides a wide range of WT. These WT are the best compromises between power and mass. Two WTs that are close in term of output power can present significantly differences in term of mass. Conversely, some WT close in term of mass can present differences in term of power. These results could be found thanks to the non-weighting of the objectives. Results also show a diversity in terms of parameters. We can have significantly different parameters WT that presents sensibly the same performances in terms of mass and power. These differences can occur between two or three of the parameters. This results in a great flexibility for both designer and manufacturer of WT.

It is possible to define other objective functions or parameters. Coupling power output depending on wind power with the probability distribution of a wind speed to occur would provide the AEP for each design of WT. In a context of performance depending on the incident wind, control actions of the WT such as load control and/or pitch angle control can be implemented in the algorithm to ensure an optimal AEP. This approach may be subject of future work.

Acknowledgement

The Region SUD (France) supported this work. The authors gratefully acknowledge this support.

References

[1] Roberto Turconi, Alessio Boldrin, Thomas Astrup, Life cycle assessment (LCA) of electricity generation technologies: Overview, comparability and limitations, Renewable and Sustainable Energy Reviews, Volume 28, 2013, Pages 555-565, ISSN 1364-0321, https://doi.org/10.1016/j.rser.2013.08.013.

[2] Jureczko, M. & Pawlak, Mariusz & Mężyk, Arkadiusz. (2005). Optimisation of Wind Turbine Blades. Journal of Materials Processing Technology. 167. 463-471. 10.1016/j.jmatprotec.2005.06.055.

[3] A.F.P. Ribeiro, A.M. Awruch, H.M. Gomes, An airfoil optimization technique for wind turbines, Applied Mathematical Modelling, Volume 36, Issue 10, 2012, Pages 4898-4907, ISSN 0307-904X, https://doi.org/10.1016/j.apm.2011.12.026.

[4] J. G. Slootweg, H. Polinder and W. L. Kling, "Dynamic modelling of a wind turbine with doubly fed induction generator," 2001 Power Engineering Society Summer Meeting. Conference Proceedings (Cat. No.01CH37262), 2001, pp. 644-649 vol.1, doi: 10.1109/PESS.2001.970114.

[5] Seul-Ki Kim, Eung-Sang Kim, Jae-Young Yoon and Ho-Yong Kim, "PSCAD/EMTDC based dynamic modeling and analysis of a variable speed wind turbine," IEEE Power Engineering Society General Meeting, 2004., 2004, pp. 1735-1741 Vol.2, doi: 10.1109/PES.2004.1373174.

[6] Wang, Lin & Liu, Xiongwei & Kolios, Athanasios, 2016. "State of the art in the aeroelasticity of wind turbine blades: Aeroelastic modelling," Renewable and Sustainable Energy Reviews, Elsevier, vol. 64(C), pages 195-210.

[7] A. Chehouri et al. "Review of performance optimization techniques applied to wind turbines," Applied Energy, vol. 142. Elsevier BV, pp. 361–388, Mar. 2015. doi: 10.1016/j.apenergy.2014.12.043.

[8] Eke, G & Onyewudiala, J. (2010). Optimization of Wind Turbine Blades Using Genetic Algorithm. Global Journal of Researches in Engineering. 10.

[9] Kevin Maki, Ricardo Sbragio, Nickolas Vlahopoulos, System design of a wind turbine using a multi-level optimization approach, Renewable Energy, Volume 43, 2012, Pages 101-110, ISSN 0960-1481, https://doi.org/10.1016/j.renene.2011.11.027

[10] B. Bavanish, K. Thyagarajan, Optimization of power coefficient on a horizontal axis wind turbine using bem theory, Renewable and Sustainable Energy Reviews, Volume 26, 2013, Pages 169-182, ISSN 1364-0321, https://doi.org/10.1016/j.rser.2013.05.009.

[11] Liu, X., Chen, Y. & Ye, Z. Optimization model for rotor blades of horizontal axis wind turbines. Front. Mech. Eng. China 2, 483–488 (2007). https://doi.org/10.1007/s11465-007-0084-9

[12] Jin Chen, Quan Wang, Wen Zhong Shen, Xiaoping Pang, Songlin Li, Xiaofeng Guo, Structural optimization study of composite wind turbine blade, Materials & Design, Volume 46, 2013, Pages 247 255, ISSN 0261-3069, https://doi.org/10.1016/j.matdes.2012.10.036.

[13] Giguere, Philippe & Selig, Michael. (2000). Blade geometry optimization for the design of wind turbine rotors. 10.2514/6.2000-45.

[14] Bottasso, Carlo & Campagnolo, Filippo & Croce, Alessandro. (2012). Multi-disciplinary constraint optimization of wind turbines. Multibody System Dynamics. 27. 21-53. 10.1007/s11044-011-9271-x.

[15] Venter, G. (2010). Review of Optimization Techniques. In Encyclopedia of Aerospace Engineering (eds R. Blockley and W. Shyy). https://doi.org/10.1002/9780470686652.eae495

[16] K. Deb et al "A Fast Elitist Non-dominated Sorting Genetic Algorithm for Multi-objective Optimization: NSGA-II," Parallel Problem Solving from Nature PPSN VI. Springer Berlin Heidelberg, pp. 849–858, 2000. doi: 10.1007/3-540-45356-3_83

[17] Yusliza Yusoff, Mohd Salihin Ngadiman, Azlan Mohd Zain, Overview of NSGA-II for Optimizing Machining Process Parameters, Procedia Engineering, Volume 15, 2011, Pages 3978-3983, ISSN 1877-7058, https://doi.org/10.1016/j.proeng.2011.08.745.

[18] Hansen, M. (2015). Aerodynamics of Wind Turbines (3rd ed.). Routledge. https://doi.org/10.4324/9781315769981

[19] Liu, S., Janajreh, I. Development and application of an improved blade element momentum method model on horizontal axis wind turbines. Int J Energy Environ Eng 3, 30 (2012). https://doi.org/10.1186/2251-6832-3-30

[20] Jin, M, Yang, X. A new fixed-point algorithm to solve the blade element momentum equations with high robustness. Energy Sci Eng. 2021; 9: 1734– 1746. https://doi.org/10.1002/ese3.945

[21] Branlard, Emmanuel. (2017). The blade element momentum (BEM) method. 10.1007/978-3-319-55164-7_10.

[22] FINGERSH, L., HAND, Maureen, et LAXSON, A. Wind turbine design cost and scaling model. National Renewable Energy Lab.(NREL), Golden, CO (United States), 2006.

[23] Garambois, Pierre & Perret-Liaudet, Joël & Rigaud, Emmanuel. (2017). NVH robust optimization of gear macro and microgeometries using an efficient tooth contact model. Mechanism and Machine Theory. 117. 78-95. 10.1016/j.mechmachtheory.2017.07.008.