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Cross-sectional temperature field of a solar collector's absorber in the case of annular pipe

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Abstract. In households, solar collectors are finding ever increasing use for hot water preparation. Although the design of solar collectors is relatively simple, the authors over many years concentrate attention on modelling the cross-sectional temperature field of the solar collector's absorber [1-7].

In solving the cross-sectional temperature field for an annular pipe [1], the periodical cross-sectional domain is divided into three sub-domains where the first sub-domain is the plate between pipes, the second – a pipe's wall, and the third – the liquid; as a result, the temperature field expressions have been obtained for all the three sub-domains. In works [2, 3] the temperature fields obtained were simplified. To define its variations in time, the temperature field was found by solving the Laplace equation under non-stationary time-dependent conditions [4]. The obtained results evidence that the non-stationary conditions might not be taken into account in long-lasting sunny weather, while such non-stationarity changes considerably the temperature field when it is short-term (e.g. in cloudy weather). The temperature has also been found for a square-pipe absorber [7] as more technological in design.

In the present work, the final temperature field model is proposed for the absorber of a round-pipe collector. As distinguished from [1], in this work the temperature of liquid is assumed to be constant over the entire pipe cross-section. Such an assumption significantly simplifies the calculation while not changing the physical essence.

Key words

Solar collectors, absorber, cross-sectional temperature field, temperature field model, annular pipe.

1. Problem formulation and temperature field solution

The shape of absorber for which the temperature field is sought is shown in Fig.1.

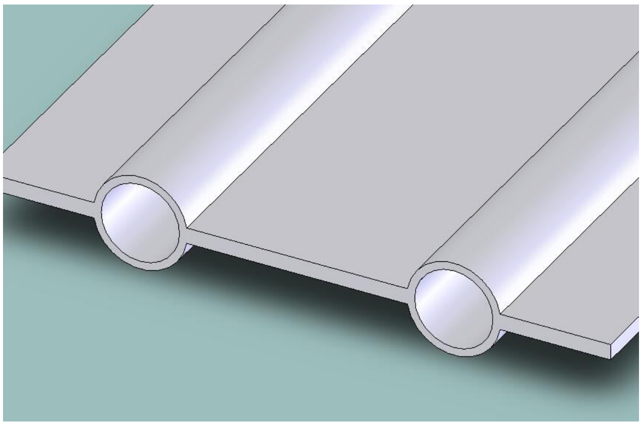

Fig 1. Absorber shape for which the temperature field is sought.

Assuming that the process is stationary, the temperature field is described by the Laplace equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0\,, \tag{1}$$

To solve this equation, the boundary conditions are set for the collector absorber's cross-section. The cross-section possesses periodicity, and the periodical domain is divided into two parts, D1 and D2. In D2 part the Laplace equation is written in polar coordinates.

On the Sun-oriented surface the incidence of solar radiation is perpendicular to all its points except the pipe surface, for which the perpendicular radiation component is calculated. The area of periodical cross-section and the boundary conditions are shown in Fig. 2. The bottom part is insulated, therefore, the heat flow perpendicular to the insulated surface is absent. The same situation is for the symmetry axes AO_1, GF and DE. On the Sun-facing surface the heat flow is proportional to the solar radiation density. In turn, on the surface where the cooling liquid meets internal wall of a pipe the heat flow is described by a typical heat flow equation where T_s is the liquid temperature and T is the temperature of a pipe's inside wall.

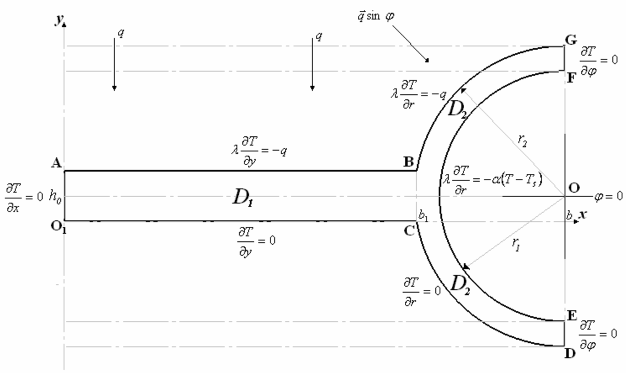

Fig 2. Absorber cross-sections with the set boundary conditions

To simplify the analytical solution of Laplace's equation we will pass to the dimensionless parameters:

$$\xi = \frac{x}{b}; \xi_1 = \frac{b_1}{b}; \xi_2 = \frac{b_2}{b}; \begin{pmatrix} 0 \le x \le b; \\ 0 \le \xi \le 1; \end{pmatrix}, \tag{2}$$

$$\eta = \frac{y}{b}; \eta_0 = \frac{h_0}{b}; \begin{pmatrix} -r_2 \le y \le r_2; \\ -\rho_2 \le \eta \le \rho_2; \end{pmatrix} \tag{3}$$

$$\rho = \frac{r}{b};\ \rho_1 = \frac{r_1}{b};\ \rho_2 = \frac{r_2}{b}; \begin{pmatrix} r_1 \le y \le r_2; \\ \rho_1 \le \rho \le \rho_2; \end{pmatrix}. \tag{4}$$

The parameters are defined by the formulas:

$$Q = \frac{q}{\lambda}\frac{b}{T_0};\ Bi = \frac{\alpha}{\lambda} b, \tag{5}$$

The dimensionless temperatures are written as:

$$\Theta_1 = \frac{T_1}{T_0}; \Theta_2 = \frac{\Delta T_2}{T_0}, \tag{6}$$

where

$$\Delta T_2 = T_2 - T_s, \tag{7}$$

The designations in the above formulas are as follows:

2b is the interval between the pipe axes, m;
$2h_0$ is the thickness of the collector's plate, m;
r_1 is the internal radius of the pipe, m;
r_2 is the external radius of the pipe, m;
q is the solar heat density, w/m^2;
λ is the thermal conductivity of the collector's, w/m K;
α is the coefficient of convective heat transfer from the surface, w/m^{2}K;
T_s is the temperature of liquid, [K];
T_0 is the initial temperature, [K].

Having written the boundary conditions in dimensionless parameters and solving the Laplace equation we obtain the following expression for the temperature field in domain D1:

$$\Theta_1(\xi,\eta) = \frac{Q}{4\eta_0}\left(\eta^2 - \xi^2\right) + F, \tag{8}$$

where

$$F = 1 - Q\eta_0. \tag{9}$$

The temperature field in domain D2 is sought-for in polar coordinates, with its dimensionless form written as

$$\Theta_2(\rho,\phi) = \frac{Q}{\pi}\left(\frac{1}{Bi\rho_1} + \ln\frac{\rho}{\rho_1}\right) - \frac{Q}{\pi\eta_0}\sum_{k=1}^{\infty}\frac{\rho}{k}\left(\frac{\rho}{\rho_2}\right)^{k-1} \bullet$$

$$\bullet \frac{1+\left(\frac{\rho_1}{\rho}\right)^{2k}\frac{k-Bi\rho_1}{k+Bi\rho_1}}{1-\left(\frac{\rho_1}{\rho_2}\right)^{2k}\frac{k-Bi\rho_1}{k+Bi\rho_1}}\cos\frac{k\pi}{2} \bullet$$

$$\bullet\left\{\eta_0\left[\frac{\cos(k-1)\varphi_2}{k-1} - \frac{\cos(k+1)\varphi_2}{k+1}\right]\left[\frac{\sin(k+1)\phi_2}{k+1} + \frac{\sin(k-1)\phi_2}{k-1}\right] + \right.$$

$$\left. + \rho_2\left[\frac{\sin(k+2)\phi_2}{k+2} + \frac{\sin(k-2)\phi_2}{k-2}\right]\right\}\cos k\left(\varphi - \frac{\pi}{2}\right), \tag{10}$$

where $\left(\rho_1 \le \rho \le \rho_2;\ \frac{\pi}{2} \le \varphi \le \frac{3\pi}{2}\right)$.

2. Results and Discussion

Fig. 3 shows the temperature field on the collector surface with the parameters given in Table 1 and at different *b* (i.e. distance from the plate centre to the pipe centre) values. The temperature in the direction from the plate centre to the pipe is decreasing – as might be expected since the liquid flowing through the pipe has a lower temperature than in the plate, and this difference creates a temperature gradient in the direction of which the heat is flowing.

Table 1. Output data for the model

Plate thickness, h [m]	Pipe external diameter, [m]	Pipe internal diameter, [m]	Solar radiation density, q [W/m^2]	Plate thermal conductivity, λ [W/mK]	Convective heat transfer, α [W/m^{2}K]	Initial temperature [K]
0.001	0.012	0.01	1000	385	1000	373

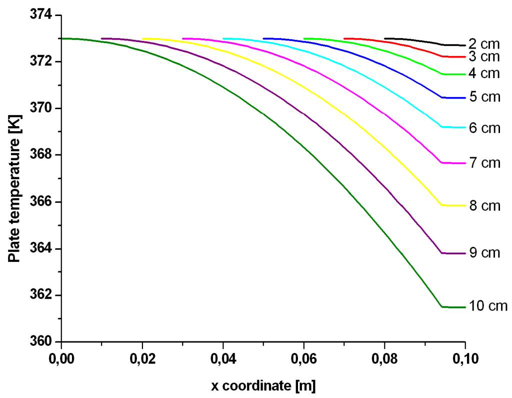

Fig. 3. Temperature field at different *b* values (the parameters are given in Table 1).

Fig. 4 shows the temperature field in the pipe coating at variable b value. As compared with temperature variation in the plate (Fig. 3), in the coating it is much less, but in any case at increasing b the temperature difference between the top of pipe and the point of its contact with the plate increases.

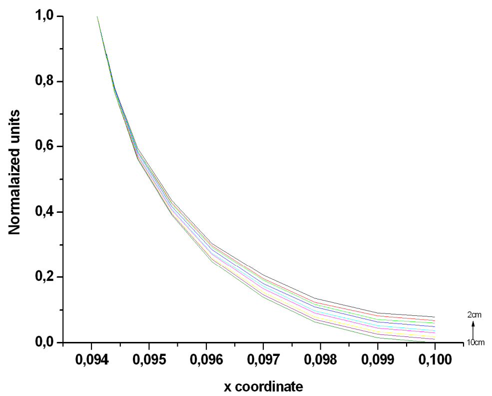

Fig. 4. Coating surface temperature variations at different b values.

In Fig.5 the temperature of liquid is shown in dependence on b value for different outside diameters of the pipe and a constant thickness of its wall (1mm). At the same pipe diameter, as the b value is increasing the temperature of liquid decreases. In physical terms this is explainable with the fact that the temperature gradient should be in the whole cross-section periodical system as implied by the temperature field mathematical expressions obtained. If the distance to the lowest temperature point is increasing, the difference between the initial temperature and the lowest one (the temperature of liquid in the proposed model) also increases.

It is seen that at increasing the pipe radius the mentioned difference as compared with that for the previous radius becomes smaller, which indicates that there is such a critical value for the diameter after which it is of no use to increase it further.

Dependence of the liquid temperature on the pipe diameter at a constant b value (0.07m) is illustrated in Fig. 6, where the curve has a saturated character. It could be seen that at the diameter value of 12 mm the liquid temperature is changing but slightly with increasing diameter. In principle, this curve shows how the pipe diameter should be optimized.

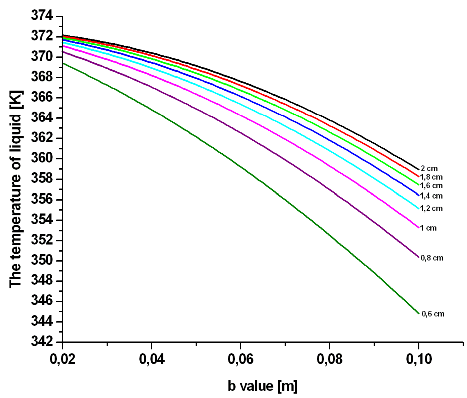

Fig. 5. The temperature of liquid vs. b value at different diameters of the pipe with a constant thickness of its wall (1mm).

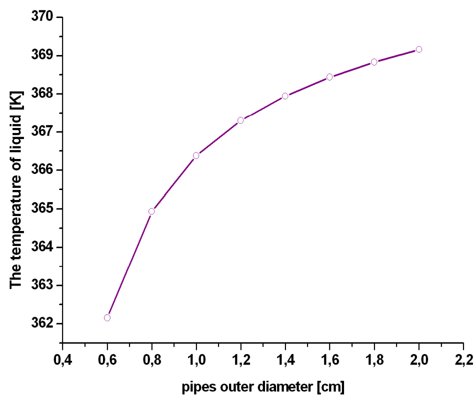

Fig. 6. Illustration of the pipe diameter variation influence on the liquid temperature.

The liquid temperature dependence on b value at different plate thicknesses is shown in Fig. 7 (the initial parameters are as given in Table 1, the pipe diameter is 12mm, the pipe wall is 1mm thick). At increasing plate thickness the dependence of the liquid temperature on the initial temperature weakens in the same manner as it is for the case with increasing radius (see Fig. 5).

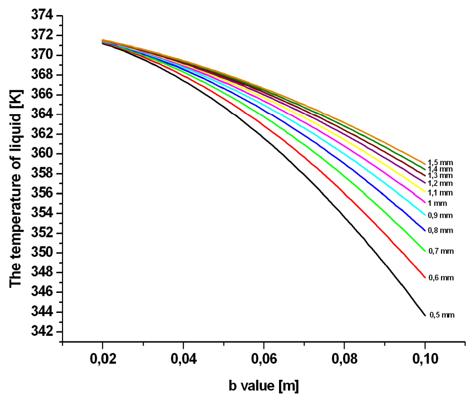

Fig. 7. Dependence of the liquid temperature on b value at different plate thicknesses (h).

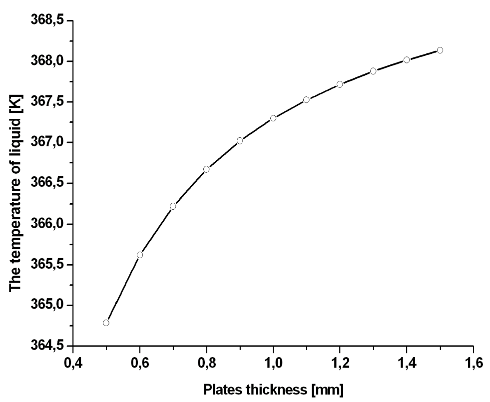

Fig. 8. Liquid temperature *vs*. plate thickness (h).

The curve shown in Fig. 8 (the dependence of liquid temperature on the plate thickness) also possesses a saturated character, which evidences that a threshold value of the plate thickness exists after which it is of no special sense to increase it further.

The basic question to be discussed is: which is better – when the liquid temperature difference from the initial temperature is large or when it is small? If this difference is large (i.e. b value is large, the pipe diameter and plate thickness are small), the heat flow from the plate centre to the pipe will be large, since this flow is directly proportional to the temperature gradient. In turn, if this difference is small (i.e. b value is small while the pipe diameter and the plate thickness are large), the heat flow will be weaker. In any case of importance is that the heat flow be large; at the same time, if b value is large and the pipe diameter is small, the amount of liquid per area unit will also be small, which would mean less per area unit power for the solar collector. Clear enough that the power decrease will be compensated at some time moment by a heat flow increase.

The proposed mathematical model shows that in all the cases discussed that the plate thickness and the pipe diameter possess threshold values, so it is useless to increase them further.

3. Conclusion

In the work, the cross-sectional temperature field has been obtained for the solar collector's absorber under the assumption that the heat flow is stationary. The results clearly show the temperature variations in the absorber's cross-section while inducate minor fluctuations the cause of which is of mathematical origin rather than of physical. The dependence of liquid temperature on different geometrical parameters points to existence of optimal values that are not to be increased since this would not affect this temperature.

Under the conditions when solar radiation is strong and constant for a long time, it is expedient to make the collector's absorber with a small b value (the distance from the plate centre to the pipe centre), a large pipe diameter (up to 14mm) and a large plate thickness (up to 1.2mm); at the same time, when cloudiness dominates, it would be better to design the solar collector with a greater b, a smaller pipe diameter (from 6 to 8 mm) and a small plate thickness (0.5mm).

Acknowledgement

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