

Combinatorial optimization for electric vehicles management

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January 11, 2011

Abstract

Growing concerns about environmental quality of cities are calling for sustainable road transportation technologies. Electric Vehicles (EV), for public and private transport, can contribute significantly to the lowering of the current pollution levels. However, the EV use is currently facing several weaknesses among which are: limited driving range, high cost and overall limited efficiency. This paper aims at specifying some key contributions of combinatorial optimization for an efficient electric vehicles management.

Electric vehicles management, Routing problem, Facility location, Vehicles redistribution.

1 Introduction

Distribution and transportation systems have been intensively studied in the operations research literature [17]. 73% of all oil consumed in Europe is used in transport and road transport accounts for 25% of CO₂ emissions of the overall transport activity. From both an environmental and energy points of view, the introduction of EV should be a first priority for the reduction of primary energy consumption.

Although higher concerns are the opportunities EV provided in terms of efficiency and flexibility in the use of energy, the EV use however is currently facing several weaknesses among which are: (1) The low energy density of batteries compared to the fuel of combustion engines, (2) EV often have long recharge times compared to the relatively fast process of refueling a tank and (3) The scarcity of public charging stations.

Electric Vehicles Management (EVM) is a relatively

recent problem, its purpose is to expedite the establishment of a customer convenient, cost-effective, EV infrastructure. In spite of the relevance of the problem, a few small research communities in this field work on some aspects of this problem. In this work, we discuss some important issues of this problem and show how CO tools can be used for solving some challenging subproblems.

Routing of EV is a major aspect of EVM, it consists of designing routes for maximizing the autonomy of vehicles, efficient EV routing plays a major role for encouraging EV use. We discuss in this paper this problem, we present a mathematical formulation of the *energy shortest path problem* and the *energy routing problem* and we expose some relationships between these problems and other well-known routing problems.

Limited driving distance between battery charges is a fundamental obstacle to broad consumer adoption of EV. In order to eliminate this fundamental disadvantage and increase consumer acceptance and usage of EV, a sufficient number of charge stations is required. The objective is to establish a charging network that is conveniently placed in familiar places to meet consumers' needs. The localization of EV charge stations is known in CO field as the *facility location problem*, we define here this problem and we expose some models proposed in the literature.

The Self-Service Electric Vehicles (SS-EV) is a key concept for developing urban clean mobility. The "free" use of EV would cause either overflow or shortage of vehicles at some stations at some times of the day. This system requires a redistribution of EV over the stations, this problem is generally modeled as a special *pick-up and delivery problem*, we discuss in this paper some of its characteristics and resolution meth-

ods.

The remainder of this paper is as follows. In section 2 we discuss routing of EV and its relationships with some routing problems in the literature. In section 3 we address the charge stations localization. We expose redistribution of EV in the context of a SS_EV system in section 4 and we conclude this paper in section 5.

2 Routing

The rising and highly-variable cost of fuel, increase the importance of efficient vehicle routing. Key advantages of EV are their ability to recover braking energy to be restored to the battery (regenerative braking) and their zero energy consumption in congested environment. Efficient EV routing is critical to the operational profitability and customer satisfaction of EV use, especially in light of highly concurrence with fuel of combustion engined vehicles.

2.1 Energy shortest path problem

The Energy Shortest Path Problem (EnSPP) consists of finding an optimal origin-destination route for EV with rechargeable batteries taking into account energy recuperation during deceleration phases. This problem can be modeled with a directed graph $G = (N, A)$, where $N = M \cup \{s, t\}$, M is the set of nodes representing the junctions, s and t the source and destination nodes respectively and A represents the set of arcs. With each arc (i, j) is associated a positive (resp. negative) value e_{ij} indicating consumption (resp. gain) of energy on arc (i, j) . The battery of each vehicle has a maximum capacity C and cannot be discharged below zero. The EnSPP consists of finding optimal origin-destination routes for EV by maximizing the vehicles battery charge at the destination node.

Very scarce works interest to this problem in the literature. In [1], the authors formalize the EnSPP problem as a generalization of the shortest path problem with hard constraints (which impose that the battery cannot be discharged below zero) and soft constraints (which impose that the battery cannot store more energy than its maximum capacity). A generic shortest path algorithm was proposed to solve the problem. The authors developed a prototypic software system for energy efficient routing where data is obtained by combining geospatial data (of OpenStreetMap) and elevation data (of the NASA Shuttle Radar Topographic Mission).

The formulation of the EnSPP is given as follows:

$$\min \quad u_t \tag{1}$$

$$\text{s.c.} \quad \sum_{i \in M} x_{si} = \sum_{i \in M} x_{it} = 1 \tag{2}$$

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = 0, \forall i \in M \tag{3}$$

$$x_{ij}(u_j - u_i - e_{ij}) \geq 0, \forall (i, j) \in A \tag{4}$$

$$0 \leq u_i \leq C, \quad \forall i \in N \tag{5}$$

$$x_{ij} \in \{0, 1\}, \quad \forall (i, j) \in A \tag{6}$$

$$u_i \in \mathbb{N}^+, \quad \forall i \in N \tag{7}$$

where variables x_{ij} indicate whether arc (i, j) is used or not and u_i is the battery remaining storage capacity at node i . Constraints (2)-(3) define the path structure for the vehicle. Energy constraints (4) express the fact that if arc (i, j) is used, the battery remaining storage capacity at node j can be greater than $u_i + e_{ij}$ (in the case where $u_i + e_{ij} < 0$).

The EnSPP is a generalization of the shortest path problem with time windows [6] where e_{ij} represents the travel time on arc (i, j) and the time window at each node is equivalent to $[0, C]$. Hard and soft constraints are equivalent to time windows constraints; Hard constraints express the fact that a vehicle must arrive at the costumer before the end of time window and soft constraints express the fact that if a vehicle arrives before the beginning of the time window, it wait at no cost. In a more general case [1], the EnSPP is a generalization of the Shortest Path Problem with Resource Constraints (SPPRC) [10] where resource consumption represents energy consumption, the amount of available resource is equal to C , and the residual resource at node i is equivalent to u_i .

2.2 Energy vehicle routing problem

The Vehicle Routing Problem (VRP) [17] is an important problem in the fields of transportation, distribution and logistics. It can be modeled with the graph G described at section 2.1, where nodes represent costumers. A set K of identical vehicles are available, each one has a maximum capacity Q . The VRP consists of finding a set of minimum cost origin-destination routes, such that each costumer $i \in M$ is visited by exactly one vehicle to satisfy a specific demand d_i . The total customers demand satisfied by the same vehicle must not exceed the vehicle capacity.

Distribution activities cause major problems with regard to noise and air pollution, so, there is a need for EV in urban distribution activities. We introduce here the VRP where vehicles are electric ones, called Energy Vehicle Routing Problem (EVRP), its formulation is given as follows:

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$$\min F(x, u) \quad (8)$$

$$\text{s.c.} \quad \sum_{k \in K} \sum_{j \in M} x_{ij}^k = 1, \forall i \in N \quad (9)$$

$$\sum_{(i,j) \in A} d_i x_{ij}^k \leq Q, \forall k \in K \quad (10)$$

$$\sum_{i \in M} x_{si}^k = \sum_{i \in M} x_{it}^k = 1, \forall k \in K \quad (11)$$

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = 0, \forall i \in M, \forall k \in K \quad (12)$$

$$x_{ij}^k (u_j^k - u_i^k - e_{ij}) \geq 0, \forall (i, j) \in A, \forall k \in K \quad (13)$$

$$0 \leq u_i^k \leq C, \forall i \in N, \forall k \in K \quad (14)$$

$$x_{ij}^k \in \{0, 1\}, \forall (i, j) \in A, \forall k \in K \quad (15)$$

$$u_i^k \in \mathbb{N}^+, \forall i \in N, \forall k \in K \quad (16)$$

where x_{ij}^k indicate whether arc (i, j) is used or not by vehicle k and u_i^k is the battery remaining storage capacity of vehicle k at node i . The objective function (8) can be considered as the sum of the remaining battery storage capacity of all vehicles ($\sum_{k \in K} u_t^k$) or the maximum remaining battery storage capacity of all vehicles ($\max_{k \in K} u_k^k$) at the destination node. Constraints (9) ensure that each customer is visited exactly once. Constraints (10) ensure that demand of each route is within the capacity limit of the vehicle serving the route. Constraints (11)-(12) are path constraints. Constraints (13)-(14) ensure compatible remaining storage capacity at each node for each vehicle.

As showed in section 2.1, the EVRP can be considered as a generalization of the vehicle routing problem with time windows [11] where the objective is to minimize the total travel time of all vehicles or the minimization of the makespan (minimization of the maximum total travel time). Additionally, this problem is a special case of a much studied problem in the literature, the Pick-up and Delivery Problem (PDP) [15], where pick-up are not hard constraints: partial pick-ups are allowed and pick-up is not performed when the vehicle is full. Acceleration (resp. deceleration) phase can be considered as a delivery (resp. pick-up) operation.

Lets $\hat{G} = (\hat{N}, \hat{A})$ be a directed graph where \hat{N} is the set of nodes and \hat{A} is the set of arcs. \hat{G} is constructed using the graph G as follows: let be $P(i)$ the set of predecessor nodes of node $i \in N$, $|P(i)| = l^i$. Each node $i \in N$ is duplicated into l^i nodes $\hat{N}_i = \{i^1, \dots, i^{l^i}\}$, so, $\hat{N} = \bigcup_{i \in N} \hat{N}_i$. For each new node, we assign an incoming arc from one predecessor node:

$$\hat{A} = \bigcup_{i \in N} \{\hat{A}(j^1, i^1) \cup \hat{A}(j^2, i^2) \dots \cup \hat{A}(j^{l^i}, i^{l^i}) \mid j^1, \dots, j^{l^i} \in P(i)\}.$$

where $A(j^1, i^1) = \bigcup_{k \in \hat{N}_{j^1}} \{(k, i^1)\}$. For each node $i \in N$, we associate to the duplicated nodes i^1, \dots, i^{l^i} the weights $q^1 = e_{j^1 i}, \dots, q^{l^i} = e_{j^{l^i} i}$. Node $i \in \hat{N}$ represents a pick-up (resp. delivery) node if $q_i < 0$ (resp. $q_i > 0$).

Optimizing the vehicle braking routes, through maximization of final remaining energy, can result in an increase of travel time. To make energy vehicle routing effective, the travel time has to be taken into account in the optimization model. As our knowledge, no work considers the travel time in energy routing problem. Such multiobjective problem can be solved using adaptation of multiobjective routing methods [12, 16].

In practice, the road links have different combinations of energy Consumption/Recuperation (C/R) levels and delays, associated with road furniture such as traffic lights and roundabouts, and road topography and geometry such as inclines. This causes energy C/R variations (resulting from acceleration and deceleration) over links with the same road category and distance. Therefore, instead of constant values, more realistic considerations of energy C/R have to be established. On one hand, energy C/R can be a function of speed variations over links. A number of works in the literature interest to model/estimate travel speed [9], these works can be exploited to estimate energy C/R. On another hand, energy C/R has to be considered as stochastic instead of constant values for assessing the risk of running out of energy before arriving at the destination. No work in the literature tackle these issues yet.

3 Facility location

One of major barriers to EV success have arisen in limited number of refueling stations, due to the limited range of EV, the establishment of an infrastructure to facilitate EV refueling is a pressing concern. Due to the large capital costs involved in infrastructure investment, economic factors are very important in determining the number and location of stations. Therefore, studies must work to provide a theoretical basis for station deployment, such as with a facility location model, to economically and efficiently serve EV trips.

Location problems in general are spatial resource allocation problems dealing with one or more service facilities serving a spatially distributed set of demands. The objective is to locate facilities to optimize a spatially dependent objective like the minimization of average travel time or distance between demands and facilities. The most studied practical problem in this context concerns hydrogen station location. In [14], general criteria are proposed for identifying effective locations for early hydrogen stations, (1) close to ar-

eas with high traffic volume, (2) in places to provide fuel during long distance trips, (3) at high profile locations to increase public awareness, and (4) in places that are accessible to individuals who are buying their first fuel-cell vehicle. These criteria are also necessary in EVM to ensure consumer confidence in the reliability of the refueling network.

In order to develop an overall broad perspective on the facility location literature so that we can appropriately model EV specific problem we present below three basic models categories that are related to refueling-station location problem [18]:

Models based on the maximum covering location problem: In [8] was introduced the “flow capturing location model” to represent the goal of locating facilities to serve passing flows. Any flow that uses a path that passes through the location of the facility was considered to be captured. The problem is formalized as a maximum covering location problem, it consists of locating p facilities so as to “capture” as many of passing flows as possible. This model was extended in [13] to the “flow refueling location model” for considering a flow refueled only if an adequate number of stations are spaced appropriately along the path.

Models based on the set covering problem: In [20], the author propose a facility location model for economically site slow recharging stations at scenic spots in order to conveniently serve all demand from single origin-destination journeys (via multi-stop refueling), with drivers using electric scooters at the destination area. This model was extended in [21] for accommodating multiple origin-destination trips. The purpose of the proposed models is to optimally site the refueling stations to cover overall passing flows on the paths of interest.

Models based the maximum covering/shortest path problem: This model [3] consists of the minimization of the path length between a given origin-destination pair of nodes and the maximization of the total demand covered by the facilities located at the nodes in the path. This model is extended in [2] to accommodate multiple origins and destinations.

4 EV redistribution

A SS_EV system represents an efficient alternative to use the private (petroleum fuel) vehicles in terms of resource sharing, cost and flexibility. This concept is currently very popular in many countries and should bring two advantages: a net reduction in the number of vehicles (and therefore parking spaces), and a reduction in air and noise pollution with the use of EV. In this system, a customer can pickup one vehicle in a station, use it for a while and return it to another (or the same) station. Indeed, either an overflow or a

shortage of vehicles can happen at one or more stations at some times of the day. This system must guarantee the availability of the vehicles at the pick-up points.

Redistribution problem consists of redistributing the vehicles among the stations in order to maximize their availability to the costumers. Redistribution is provided by a fleet of limited capacity tow-trucks located at various depots on the network. This problem can be naturally modeled as a pickup and delivery problem. In [7], this problem is viewed as a generalization of the pickup and delivery problem, formulated as a mixed integer problem and solved using a number of resolution methods including constraint programming, Lagrangian relaxation and a modified A^* heuristic. The problem seems to be difficult to solve although for small instances, this is due to the splitup of pickups and deliveries, the small capacities of tow-trucks and non simple paths (a tow-trucks can return to a station already serviced).

In [5], the authors propose a balancing technique which consists of switching from an unfavorable state to a favorable one. A favorable state is defined as the distribution of vehicles in the stations that guarantees that the system can reach a given large horizon with the highest probability, assuming that no balancing action is conducted, and an unfavorable state is defined as the distribution of vehicles in the stations that guarantees that at least one of the station will run out of vehicles or will be overload before a given small horizon with a probability greater than or equal a given threshold. In [4], the authors consider the issue of recharge problem where an optimal level of charge that makes the vehicle available for costumers is defined. Because of the battery limited range, two major aspects have to be taken into account in the redistribution system, the physical and energy availability of vehicles at stations.

5 Conclusion

The succeeding of a transition from a conventional gasoline based transportation system towards a sustainable way of transportation, depends on a quite number of critical factors. The substitution of conventional vehicles through electric and/or hybrid vehicles involves economical, environmental and social aspects. The purpose of EVM is to encourage the transition to EV use and to expedite the establishment of a convenient, cost-effective, EV infrastructure that such a transition necessitates. Whereas the development of EV through battery autonomy grow extensively, very little research is dedicated to EV routing, recharge stations localization and vehicles redistribution. We discussed in this paper these major aspects which represent challenging optimization problems.

Routing consists of designing routes for EV for max-

imizing the autonomy of vehicles, efficient EV routing plays a major role for encouraging EV use. This problem is much close to a number of well-studied routing models in the literature.

Compared to fossil fuels, batteries have a low energy content per weight ratio. This limits the radius of action of EV, making them mostly suitable for urban traffic and special transport applications, such as shuttle services. Siting sufficient refueling stations along intercity highways is central issue to ensure the completion of the long-distance travel demands in order to foster the use of EV.

There is an increase on utilization of video surveillance, global positioning systems and communication equipment installed on vehicles. A topic of interest is the development of a global mobility information systems through which EV charging station customers can locate EV charging stations prior to starting on a trip, and can maximize their mileage and minimize their risk of running out of electricity.

Problems discussed in this paper are subproblems of EVM that can be treated using combinatorial optimization tools. The global critical problem in the EV promotion is a chicken-and-egg infrastructure dilemma [19]: consumers will be reluctant to purchase vehicles until a sufficient number of refueling stations has been installed, while vehicle manufactures will not produce vehicles that consumers will not buy, and fuel providers will not invest in a new energy infrastructure until there is sufficient demand for it. Therefore, it is likely that governments will need to play a significant role in promoting any change to alternative fuels, although public support alone will not ensure the success of this transformation.

Summarizing the paper discussions, it can be stated that EVM systems including the three topics discussed in this paper are a crucial path towards EV mobility. The key functionalities assigned to such a system are gain of driving range, the flexibility of recharging vehicles and vehicles availability in a self-service EV system.

References

- [1] A. Artmeier, J. Haselmayr, M. Leucker and M. Sachenbacher, The optimal routing problem in the context of battery-powered electric vehicles, Workshop: CROCS at CPAIOR-10, Second International Workshop on Constraint Reasoning and Optimization for Computational Sustainability, Bologna, Italy, 2010.
- [2] R. Bapna, L.S. Thakur and S.K. Nair, Infrastructure development for conversion to environment friendly fuel, *European Journal of Operational Research*, 142(3):480-490, 2002.
- [3] J.R. Current, C.S. ReVelle and J.L. Cohon, The maximum covering/shortest path problem: a multiobjective network design and routing formulation. *European Journal of Operational Research*, 21(2):189-199, 1985.
- [4] C. Fabrice, N. Hafez and J-M. Proth, Electric vehicles: Effect of the availability threshold on the transportation cost, *Applied stochastic models in business and industry*, 15(3):169-181, 1999.
- [5] C. Fabrice, N. Hafez, J-M. Proth and N. Sauer, Management of a pool of self-service cars, *Journal of Intelligent Manufacturing*, 8(5):459-465, 1997.
- [6] G. Desaulniers and D. Villeneuve, The shortest path problem with time windows and linear waiting costs, *Transportation science*, 34(3):312-319, 2000.
- [7] M. Dror, D. Fortin and C. Roucairol, Redistribution of self-service electric cars: A case of pickup and delivery, Technical Report W.P. 3543, INRIA-Rocquencourt, Rocquencourt, France, 1998.
- [8] M.J. Hodgson, A flow capturing location-allocation model, *Geographical Analysis* 22(3):270-279, 1990.
- [9] A.V. Hill and W.C. Benton, Modeling intra-city time-dependent travel speeds for vehicle scheduling problems, *Journal of the Operations Research Society*, 43(4):343-351, 1992.
- [10] S. Irnich and G. Desaulniers, Shortest Path Problems with Resource Constraints, In G. Desaulniers, J. Desrosiers, and M.M. Solomon, editors, *Column Generation*, chapter 2, pages 33-65, Springer 2005.
- [11] B. Kallehauge, J. Larsen, O.B.G. Madsen and M.M. Solomon, Vehicle Routing Problem with Time Windows, In G. Desaulniers, J. Desrosiers, and M.M. Solomon, editors, *Column Generation*, chapter 3, pages 67-98, Springer 2005.
- [12] N. Jozefowicz, F. Semet and El-Ghazali Talbi, Multiobjective vehicle routing problems, *European Journal of Operational Research*, 189(2):293-309, 2008.
- [13] M. Kuby and S. Lim, The flow-refueling location problem for alternative-fuel vehicles, *Socio-Economic Planning Sciences* 39(2)/125-145, 2005.
- [14] M.W. Melaina, Initiating hydrogen infrastructure: preliminary analysis of a sufficient number of initial hydrogen stations in the US, *International Journal of Hydrogen Energy* 28(7),743-755.

- [15] S.N. Parragh and K.F. Doerner and R.F. Hartl, A survey on pickup and delivery problems, *Journal für Betriebswirtschaft*, 58(2):81-117, 2008.
- [16] Z. Tarapata, Selected multicriteria shortest path problems : An analysis of complexity, models and adaptation of standard algorithms, *International Journal Of Applied Mathematics And Computer Science*, 17:269-287, 2007.
- [17] P. Toth and D. Vigo, *The vehicle routing problem*, Philadelphia: Society for Industrial and Applied Mathematics, 2002.
- [18] Y.W Wang and C.C Lin, Locating road-vehicle refueling stations, *Transportation Research Part E: Logistics and Transportation Review*, 45(5):821-829, 2009.
- [19] Y.W Wang and C.R. Wang, Locating passenger vehicle refueling stations, *Transportation Research Part E: Logistics and Transportation Review*, 46(5):791-801, 2010.
- [20] Y.W Wang, An optimal location choice model for recreation-oriented scooter recharge stations, *Transportation Research Part D: Transport and Environment*, 12(3):231-237, 2007.
- [21] Y.W Wang, Locating battery exchange stations to serve tourism transport, *Transportation Research Part D: Transport and Environment*, 13(3):193-197, 2008.