Comparison of Gabor-Wigner Transform and SPWVD as tools of harmonic computation.

M. Szmajda[1], J. Mroczka[2]
[1] Faculty of Electrical Engineering, Automatic Control and Informatics
Opole University of Technology
Ul. Sosnkowskiego 31, 45-272 Opole (Poland)
Phone/Fax number: +0048 77 4006238, e-mail: **m.szmajda@po.opole.pl**,

[2] Faculty of Electronics
Wrocław University of Technology
ul. B. Prusa 53/55, 50-317 Wrocław (Poland)
Phone/Fax number: +0048 71 3206232, e-mail: **janusz.mroczka@pwr.wroc.pl**,

Abstract. The measurement algorithms applied in power quality measurement systems are based on Fast Fourier Transformation. That one-dimension frequency analysis is sufficient in many cases. However, to illustrate the character of the signal in a more comprehensive manner, it is crucial to represent the investigated signal on time-frequency plane. There are a lot of time-frequency representations (TFR) for presenting measured signal. The TFR: Short-Time Fourier Transformation (STFT), Smoothed Pseudo Wigner-Ville Distribution (SPWVD) and Gabor-Wigner Transform (GWT) are described in the paper. The ability of implementation of mentioned methods in harmonics computation according to the power quality standards were presented in the paper.

Key words

Harmonics, Short-Time Fourier Transform (STFT), Gabor-Wigner Transform (GWT), Smoothed Pseudo Wigner-Ville Distribution (SPWVD).

1. Introduction

The measurements of the power quality frequency parameters (i.e. THD factor) are currently performed with the help of FFT transformation [2][3]. In spite of high computation efficiency, the method does not give positive results during measurements of the fast spectrum changes. Therefore, current research is performed into the application of alternative methods enabling spectrum measurements and time localization.

Simultaneous localization disturbances in time- and frequency- domains may be performed with the help of time-frequency methods. Among many of time-frequency methods monitoring of power quality parameters is taken into consideration: Short-Time Fourier Transform (STFT), Smoothed Pseudo Wigner-Ville Distribution (SPWVD) and Gabor-Wigner Transform.

The SPWVD does not include characteristic distortions for Wigner-Ville Distribution (WVD) – time and frequency cross-terms. The SPWVD features loss of excellent WVD time-frequency resolution. To get better resolution than SPWVD and to avoid cross-term distortions the Gabor-Wigner Transform (GWT) has recently been proposed. Unfortunately, there are some specific circumstances, where GWT gives wrong results. The special signal's model, which includes these features in the researches was implemented.

It is interesting comparison of: standard STFT, SPWVD and GWT in measurements of power supply harmonics and interharmonics.

2. Power quality and standards

The power quality (PQ) issue should be regarded in relation to: EN 50160 [1] and EMC standards 61000 [2,3] family. Moreover, in many countries, there also exist local regulations defined by governmental order.

The standard [1] defines the main voltage parameters and their permissible deviation ranges at the customer's point of common coupling in public low voltage (LV) and medium voltage (MV) electricity distribution systems, under normal operating conditions.

Recommendations included in the standard characterize PQ with the help of parameters describing: power frequency, voltage magnitude, shape of voltage waveform, three-phase voltage unbalance and continuity of supply. It does not yet define the measurement methods required for computation of particular parameters. Detailed definitions, measurement methods and measurement equipment construction guidelines are presented i.e. in [2,3].

The harmonics and interharmonics measurement procedure is specifically defined in [2]. The main guidelines are described below:

- measurement systems should include: input circuits equipped with anti-aliasing filters, analog-to-digital converter with sample-hold circuit, synchronization and window-shaping unit, DFT-processor,
- the time window shall be synchronized with each group of 10 or 12 cycles according to the Power system frequency of 50 Hz or 60 Hz. These recommendations define the spectrum resolution of DFT: 5Hz for 50Hz systems and 6Hz for 60Hz systems;
- sampling frequency and number of samples in a time window should be matched to minimize synchronization error, with a permissible error of 0.03%;
- the definition of indices which characterize the content of harmonics in 50 Hz systems is

presented in Fig. 1 and with the help of (1) and (2);

- the definition of indices which characterize the content of interharmonics in 50 Hz systems is presented in Fig. 2 and with the help of (3) and (4).

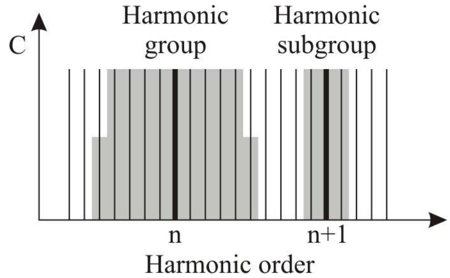

Fig. 1. Illustration of a harmonic group and subgroup.

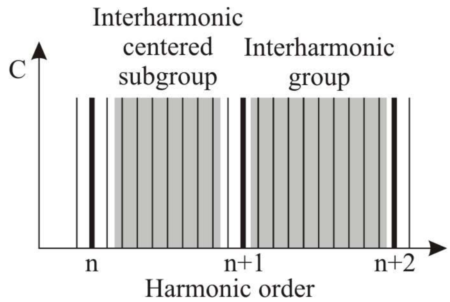

Fig. 2. Illustration of an interharmonic group and centred subgroup.

$$G_{g,n}^{2}=\frac{C_{k-5}^{2}}{2}+\sum_{i=-4}^{4}C_{k+i}^{2}+\frac{C_{k+5}^{2}}{2} \quad (1)$$

$$G_{sg,n}^{2}=\sum_{i=-1}^{1}C_{k+i}^{2} \quad (2)$$

$$C_{ig,n}^{2}=\sum_{i=1}^{9}C_{k+i}^{2} \quad (3)$$

$$C_{isg,n}^{2}=\sum_{i=1}^{8}C_{k+i}^{2} \quad (4)$$

3. Time-frequency analysis

For non-stationary signal (often encountered in power networks) research it is necessary to use time-frequency methods.

The methods can be divided in several ways. One of the divisions is shown in the Fig. 3 [4, 5].

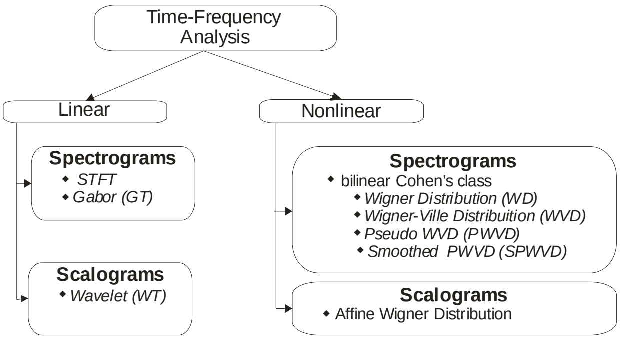

Fig. 3. One of the signal analysis classifications.

In linear methods, the signal being analyzed is compared directly with suitable elementary functions. Computational complexity of linear methods is relatively smaller in comparison to nonlinear methods.

The nonlinear methods have a significant advantage of direct energy signal projection on time-frequency plane. This is particularly important in power signal measurements. Disadvantage of linear, especially bilinear analysis, is the typical interference called cross-terms.

The so called "transformation kernel", enabling match of the analysis adaptation with the examined interference, should be properly chosen to reduce the said cross terms [5].

In the measurement of power quality parameters and power network disturbances the research is focused on application of multiple time-frequency methods that belong to the groups above. One of the most popular methods includes: Short-Time Fourier Transform (STFT), Gabor Transform (GT) and Wavelet Transformation (WT). For bilinear transformation analysis there is a research on application of i.e.: Wigner-Ville Distribution (WVD), Pseudo Wigner-Ville Distribution (PWVD), Smoothed Pseudo Wigner-Ville Distribution, Choi-Williams Distribution (CWD).

4. STFT, GWT and SPWVD in harmonics computation

Originally, continuous Short-Time Fourier Transformation (STFT) is a natural Fourier Transform extension with analyzing time-window overlay that enables to determine a point in time for signal spectrum fluctuation. Optimal adjustment of STFT parameters and results processing were the subject of many publications [6, 7, 8]. Also, guidelines for STFT analysis parameter selection were defined in standard PN-EN 61000-4-7 [2], which is currently one of the basic principles for designing electric power quality measuring devices.

Definition of such analysis is represented by formula (5) below [5]:

$$STFT(t,f)=\int_{-\infty}^{+\infty}s(\tau)\gamma^{*}(\tau-t)e^{-j2\pi f\tau}d\tau \quad (5)$$

where:

$s(t)$ - is a signal in time domain,

$\gamma(t)$ - is a signal in time-window.

In particular, substitution window $\gamma(t)$ (which is a Gauss function represented by formula (6)) to formula, (5) results (7):

$$\gamma(t)=e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^{2}} \quad (6)$$

$$GT(t,f)=\int_{-\infty}^{+\infty}s(\tau)e^{-\frac{1}{2}\left(\frac{\tau-t}{\sigma}\right)^{2}}e^{-j2\pi f\tau}d\tau \quad (7)$$

Wigner-Ville transformation (distribution) WVD is presented as follows (8) [5]:

$$WVD_s(t,f)=\int_{-\infty}^{+\infty} s\left(t+\frac{\tau}{2}\right)s^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi f\tau}d\tau \qquad (8)$$

where:

s(t) – investigated signal processed with the help of Hilbert Transformation (9):

$$\hat{s}(t)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{s(\tau)}{t-\tau}d\tau \qquad (9)$$

One of the advantages of WVD with relation to GT (and STFT also) is resolution, which is twice as high. The transformation gives very good results (high time-frequency resolution) when examined signals consist of low number of higher harmonics. In other cases, the transformation results include interferences, the so called cross-terms. They appear between each pair of harmonics and make proper disturbances interpretation impossible. It is unacceptable for analysis of disturbed power signals. Currently, researches are conducted concerning the new methods of cross-terms reduction.

Another advantage of WVD is the fact that WVD gives direct information about time-frequency localization of signal energy. It enables application of the transformation to evaluate energy included in higher harmonics and to localize it in time domain.

Equation (8) assumes limit of integration for displacement from -∞ to +∞. As a rule of a thumb, such requirement is impossible to meet, so WVD is overlaid on *h(t)* time-window, which results in Pseudo Wigner-Ville Distribution (PWVD) (as described by formula 10):

$$PWVD(t,f)=\int_{-\infty}^{+\infty} s\left(t+\frac{\tau}{2}\right)s^*\left(t-\frac{\tau}{2}\right)\cdot\ldots$$
$$\ldots\cdot h\left(\frac{\tau}{2}\right)h^*\left(-\frac{\tau}{2}\right)e^{-j2\pi f\tau}d\tau \qquad (10)$$

where:

h(t) – window reducing cross-terms in time domain.

Time-window overlay operation is equal to WVD frequency filtering. Usually, the interference between time-shifted signals is attenuated, but cross-terms between frequency-shifted signals still exist.

There are several ways of resolving the problem of frequency cross-terms attenuating. Two methods are described below.

To minimize cross-terms between components in frequency domain, PWVD results are attenuated with low-pass filtering, using the *g(t)* attenuation window. Such analysis is described as Smoothed Pseudo Wigner-Ville Distribution (SPWVD), defined with formula (11):

$$SPWVD(t,f)=\int_{-\infty}^{+\infty} g(t-t')PWVD(t',f)dt'=$$
$$=\int_{-\infty}^{+\infty} h\left(\frac{\tau}{2}\right)h^*\left(-\frac{\tau}{2}\right)\int_{-\infty}^{+\infty} g(t-t')\cdot\ldots \qquad (11)$$
$$\ldots\cdot s\left(t'+\frac{\tau}{2}\right)s^*\left(t'-\frac{\tau}{2}\right)dt'\cdot e^{-j2\pi ft}d\tau$$

where:

h(t) – window reducing cross-terms in time domain,

g(t) – window reducing cross-terms in frequency domain.

The disadvantage of using filtering windows is limiting of original excellent time-frequency resolution features.

Selection of proper *h(t)* and *g(t)* windows in expected interference function and requested spectrum resolution causes significant problems.

An alternative solution, to be used instead of SPWVD or other billing methods for cross-term reduction, is the Gabor-Wigner Transformation (GWT). GWT is defined by the following relations (12-15) [9-12], and the detailed properties are shown in [9]:

$$GWT(t,f)=GT(t,f)\cdot WVD(t,f) \qquad (12)$$

$$GWT(t,f)=\min\left\{|GT(t,f)|^2|WVD(t,f)|\right\} \qquad (13)$$

$$GWT(t,f)=WVD(t,f)\left\{|GT(t,f)|>0.25\right\} \qquad (14)$$

$$GWT(t,f)=GT^{2.6}(t,f)\cdot WVD^{0.6}(t,f) \qquad (15)$$

In fact, GWT is a composition of two time-frequency planes, being a result of Gabor Transformation and WVD. Such approach can be taken thanks to linked advantages of both transforms: excellent WVD time-frequency properties and lack of cross-terms - GT. Because of the real signal analysis it is not uncommon to use PWVD instead of WD.

5. Case studies

To demonstrate the analysis reaction on the harmonics distortion, the model of test signal was proposed (16):

$$s(t)=230\sqrt{2}\sin(2\pi 50t)+\ldots$$
$$\ldots+230\sqrt{2}\sin(2\pi 100t)+\ldots \qquad (16)$$
$$\ldots+230\sqrt{2}\sin(2\pi 150t)$$

The STFT, GWT and SPWVD analyses were examined. The input parameters of the methods are included in Table I.

Table I. – Input parameters

	Method	Parameters	Figs
	STFT	Rectangular window 0.2 s	
1	GWT	Gaussian window: 0.1 s PWVD *h(t)* window: rect. 0.1 s	10 11
	SPWVD	*h(t)* window: 0.1s, rectangular *g(t)* window:0.1s Hamming	
2	GWT	Gaussian window: 0.1 s PWVD *h(t)* window: Hamming 0.1s	12 13
	SPWVD	*h(t)* window: Hamming 0.1 s *g(t)* window: Hamming 0.1 s	
3	GWT	Gaussian window: 0.2 s PWVD *h(t)* window: rect. 0.2 s	14 15
	SPWVD	*h(t)* window: rectangular 0.2 s *g(t)* window: Hamming 0.1s	
4	GWT	Gaussian window: 0.2 s PWVD *h(t)* window: Hamming 0.2 s,	16 17
	SPWVD	*h(t)* window: Hamming 0.2 s *g(t)* window: Hamming 0.1 s	

Figs. 4-6 present time-frequency planes for STFT, GWT and SPWVD respectively (input parameters as in Table I point 4). The central sections of those images present time-frequency plane, in the lower section there is a form of a course in time, the so called frequency-marginal condition is situated on the left, while the upper section contains the time-marginal condition. For better interpretation of the results, 3D views of the analysis are shown in Figs. 7-9.

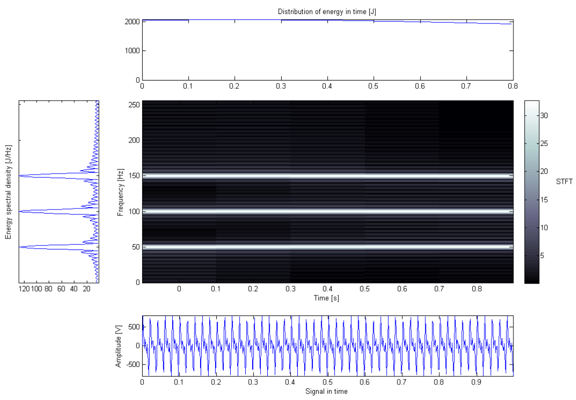

Fig. 4. STFT time-frequency plane: rectangle window 0.2 s.

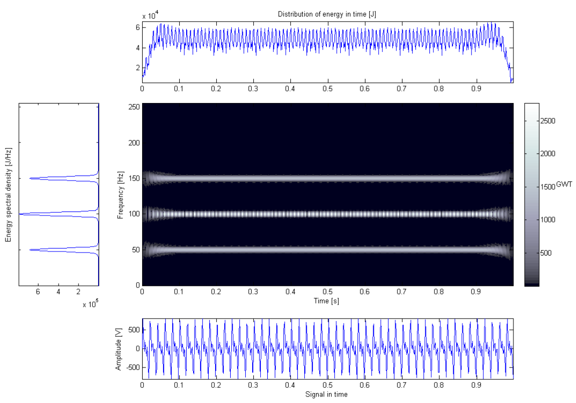

Fig. 5. GWT time-frequency plane.

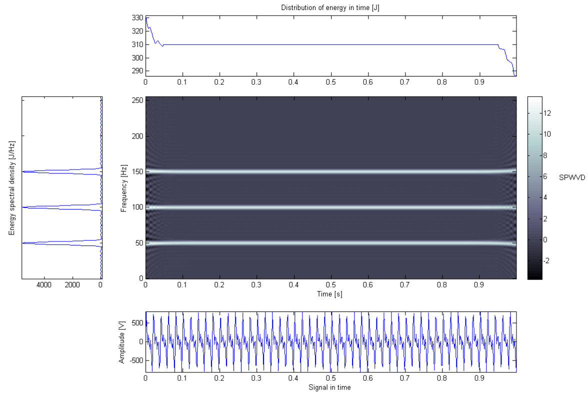

Fig. 6. SPWVD time-frequency plane.

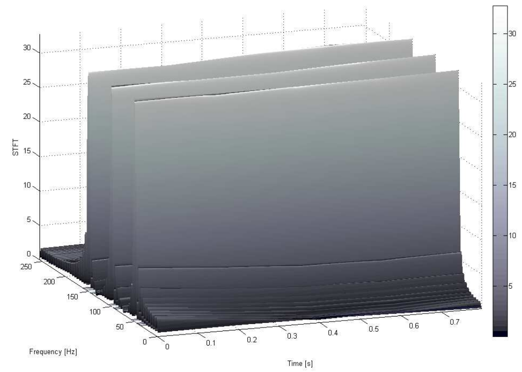

Fig. 7. 3D view of STFT time-frequency plane.

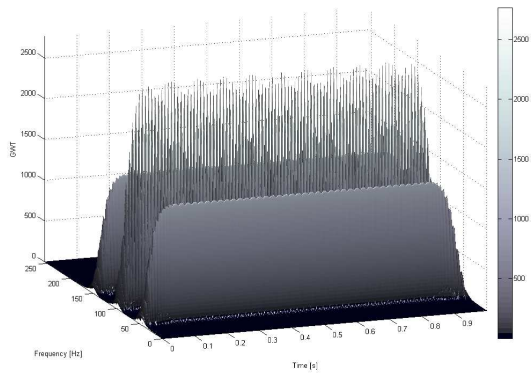

Fig. 8. 3D view of GWT time-frequency plane.

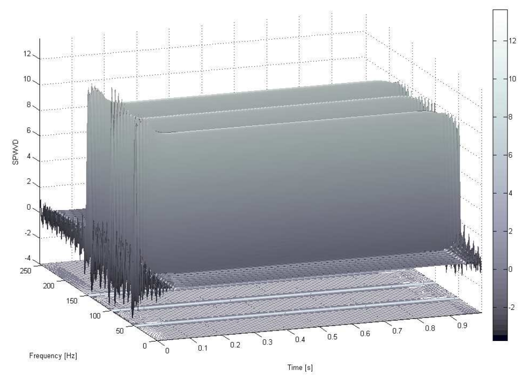

Fig. 9. 3D view of SPWVD time-frequency plane.

The GWT results of the analyzed signal show an unattenuated cross-term overlaying the 100 Hz component (Fig. 8). It is particularly apparent in Fig. 5 in frequency-marginal condition - the 100 Hz component reveals a significantly greater amplitude than other 50 Hz and 150 Hz. The 3D view (Fig. 7) shows the characteristic high-frequency interference overlaid on 100 Hz component. The interference occurs, as Gabor Transformation constituting an integral part of GWT does not attenuate the sections of signals with auto-terms. In SPWVD analysis (Figs. 6 and 9) this interference is attenuated, since all high-frequency components, including cross-terms, get reduced on time-frequency plane.

To emphasize the differences between results for individual methods, Fig. 10 to 17 show a comparison of frequency cross-sections profiles for time of 0.9 s.

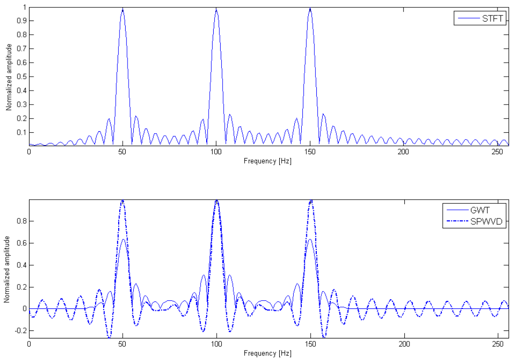

Fig. 10. Freq. profile comparison for STFT, GWT and SPWVD – linear amplitude scale (input parameters Table I point 1).

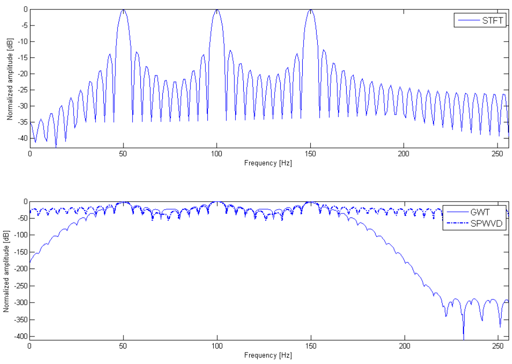

Fig. 11. Freq. profile comparison for STFT, GWT and SPWVD – decibel amplitude scale (input parameters Table I point 1).

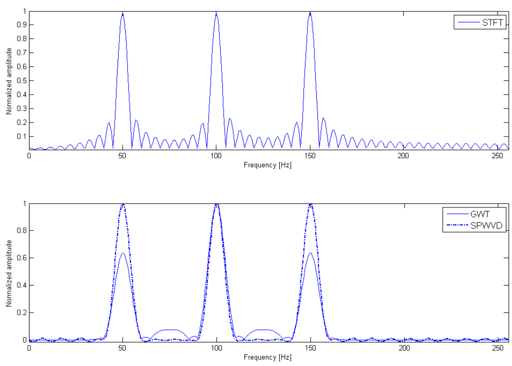

Fig. 12. Freq. profile comparison for STFT, GWT and SPWVD – linear amplitude scale (input parameters Table I point 2).

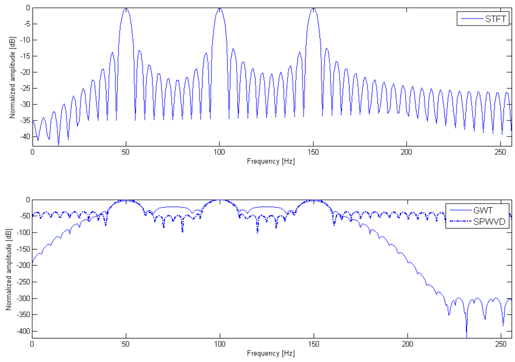

Fig. 13. Freq. profile comparison for STFT, GWT and SPWVD – decibel amplitude scale (input parameters Table I point 2).

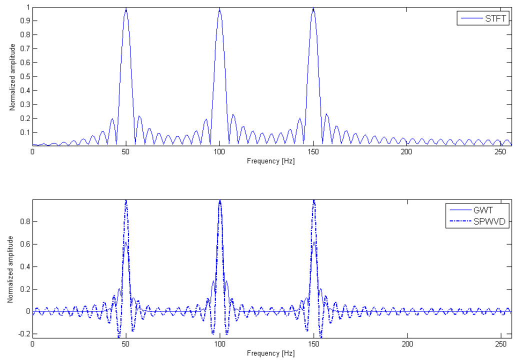

Fig. 14. Freq. profile comparison for STFT, GWT and SPWVD – linear amplitude scale (input parameters Table I point 3).

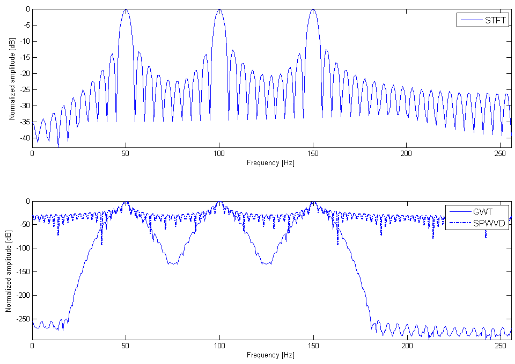

Fig. 15. Freq. profile comparison for STFT, GWT and SPWVD – decibel amplitude scale (input parameters Table I point 3).

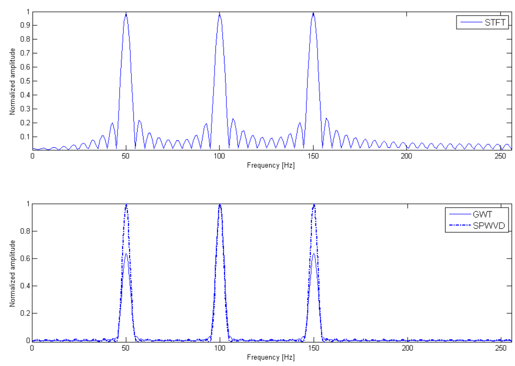

Fig. 16. Freq. profile comparison for STFT, GWT and SPWVD – linear amplitude scale (input parameters Table I point 4).

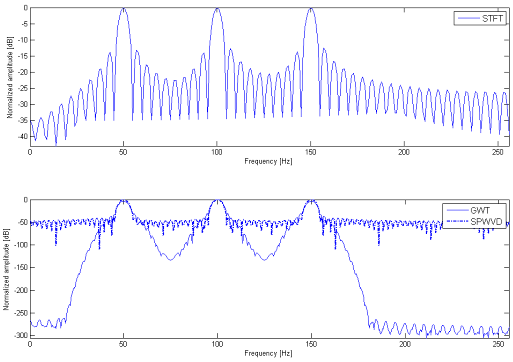

Fig. 17. Freq. profile comparison for STFT, GWT and SPWVD – decibel amplitude scale (input parameters Table I point 4).

On the basis of the analysis of the methods used, the following conclusions can be drawn:

- it is possible to replace standard STFT with SPWVD and Hamming window of 0.2 s (input parameters in Table I point 4). Such a solution guarantees the frequency resolution of 5 Hz and greater attenuation of side lobes than in case of STFT,
- application of 0.1 s Hamming window (input parameters in Table I point 2) enables measurement of interharmonics with 5 Hz resolution between main lobes. Width of main lobes equals 20 Hz. That is why it is possible of evaluating harmonics and interharmonics (i.e. considering interharmonics between basic and 2^{nd} harmonic) 50, 60, 65, 70, 75, 80, 85, 90 and 100 Hz. The broad main lobe and attenuation of side lobes about 40 dB ensure less influence of spectrum leakage phenomenon,
- GWT for the chosen signal type gives distorted results for harmonics, which are placed between two other harmonics.

4. Conclusion

Several time-frequency tools for power signal analysis have been presented above. The possibilities of their usage in detection of disturbances and higher harmonics measurement have been described. It has been proved that the GWT analysis gives invalid results for signals with frequency components whose frequency is half as much as the adjacent auto-terms. Therefore an additional analysis of time-frequency plane, aimed at eliminating these disturbances, is required.

The standard STFT analysis can be replaced with SPWVD analysis. When using Hamming window of 0.2 s in SPWVD, greater attenuation of side lobes can be obtained, as compared to STFT. Thus, the results of harmonic computation are less subject to disturbances caused by spectral leakage.

In general, SPWVD and GWT analyses, are much more computational than standard STFT. Therefore further research is conducted for the ways of their application in embedded equipment that uses these analyses in on-line computation.

Acknowledgement

This work was supported by the Ministry of Science and Higher Education, Poland (grant # N505 363336).

References

[1] EN 50160: 2007: Voltage characteristics of electricity supplied by public distribution systems.

[2] IEC 61000-4-7:2007: Electromagnetic compatibility (EMC) - Part 4-7: Testing and measurement techniques - General guide on harmonics and interharmonics measurements and instrumentation, for power supply systems and equipment connected thereto.

[3] IEC 61000-4-30:2009: Electromagnetic compatibility (EMC) -- Part 4-30: Testing and measurement techniques – Power quality measurement methods.

[4] T. P. Zieliński, „Reprezentacja sygnałów niestacjonarnych typu czas-częstotliwość i czas-skala" (Time-Frequency and Time-Scale Representation of Nonstationary Signals), Wydawnictwa AGH, Kraków, 1994.

[5] A. D. Poularikas, "The Transforms and Applications Handbook. Second Edition", CRC Press, Boca Raton 2000.

[6] M. Szmajda, K. Górecki, "DFT algorithm analysis in low-cost power quality measurement systems based on a DSP processor", 9th International Conference Electrical Power Quality and Utilisation, Barcelona (Spain), 9 – 11 October 2007.

[7] K. Górecki, M. Szmajda, "Adaptive digital synchronization of measuring window in low-cost DSP power quality measurement systems", 9th International Conference Electrical Power Quality and Utilisation, Barcelona (Spain), 9 – 11 October 2007.

[8] F. Zhang, Z. Geng, W. Yuan, "The Algorithm of Interpolating Windowed FFT for Harmonic Analysis of Electric Power System", IEEE Transactions On Power Delivery, vol. 16, no. 2, April 2001, pp. 160-164.

[9] S. C. Pei, J. J. Ding, "Relations Between Gabor Transforms and Fractional Fourier Transforms and Their Applications for Signal Processing", IEEE Transactions on Signal Processing, vol. 55, pp. 4839-4850, Oct. 2007.

[10] S. H. Cho, G. Jang, S. H. Kwon, "Time-Frequency Analysis of Power-Quality Disturbances via the Gabor-Wigner Transform", IEEE Transactions On Power Delivery, vol. 25, pp.494-499, Jan. 2010.

[11] M. Szmajda, K. Górecki, J. Mroczka, "Gabor Transform, Gabor-Wigner Transform and SPWVD as a Time-Frequency Analysis of Power Quality", Metrology and Measurement Systems, vol. XVII, number 3 (2010).

[12] M. Szmajda, K. Górecki, J. Mroczka, "Gabor Transform, Gabor-Wigner Transform and SPWVD as a Time-Frequency Analysis of Power Quality", 14th International Conference on Harmonics and Quality of Power, Bergamo (Italy) 26-29 September 2010.