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Analysis by State Equation of a Control Strategy for Hybrid Filter

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Abstract. A control algorithm is proposed for a three-phase hybrid power filter constituted by a series active filter and a passive filter connected in parallel with the load. It is based on the dual formulation of the vectorial theory of instantaneous reactive power, so that the voltage waveform injected by the active filter is able to compensate the reactive power and the harmonics of the load current. System state model was obtained and the system behaviour was analyzed by the state equations for each situation. The analysis developed has allowed the knowledge of the system dynamic behaviour and the stability margins. An experimental prototype has been developed, and simulation and experimental results are presented.

Key words

Harmonics, series active power filter, hybrid filter, state space.

1. Introduction

The active power filters (APFs) have been used in the last years to eliminate the harmonic distortion in electrical systems. An APF is a static compensation system based on an electronic converter with a Pulse Width Modulation (PWM) control. It may be connected in parallel or in series to the load. The shunt APF is connected in parallel to the load and it works as a controlled current source. These equipments allow the elimination of the current harmonic originated by the called current harmonic source loads [1,2]. It is the most studied configuration.

To eliminate current harmonics a shunt passive filter have traditionally been used, mainly due to their low cost and minimal maintenance requirements. As a result, this has been the adopted solution for systems with considerable power. It is possible to improve the behaviour of shunt passive filters including a series active filter in the system. This improves the compensation characteristics of the passive filter, [3]. This topology is shown in figure 1, where v_c is the voltage that the active power filter should generate to achieve the objective of proposed control algorithm.

Different strategies have been applied to this topology, according to the compensation target [4,7]. This work is focused on the analysis of a control strategy based on the dual formulation of the vectorial theory of electrical power, [8,9]. The voltage waveform injected by the active filter is able to compensate the reactive power and the harmonics of the load current. A state model of the system has been developed. This analysis has allowed the knowledge of the system dynamic. The system behaviour has been contrasted by means of a laboratory prototype and experimental results have been presented.

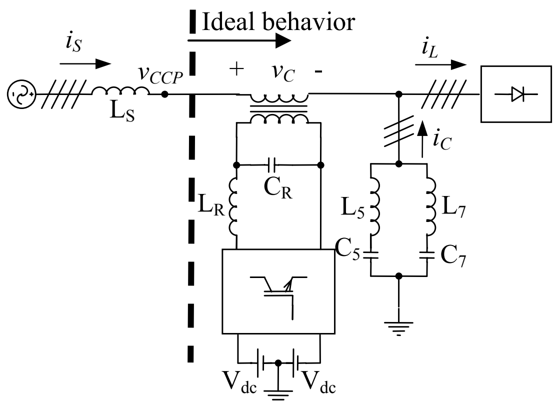

Fig. 1. Scheme of a hybrid filter, series active filter and shunt passive filter

2. Compensation Strategy

Electrical companies try to generate electrical power with sinusoidal and balanced voltages and it has been considered as a reference condition in the supply. Due to this fact, the compensation target is based on an ideal reference load which must be resistive and linear. It means that the source currents are collinear to the supply voltages (equation 1) and the system will have unity power factor.

$$\boldsymbol{v} = R_e \, \boldsymbol{i} \tag{1}$$

R_e is the equivalent resistance, $\boldsymbol{v}$ the voltage vector on the connection point and $\boldsymbol{i}$ the load current vector. When the currents are non-sinusoidal, a balanced resistive load is considered as ideal reference load, therefore, the active power supplied by the source will be

$$P_S = I_1^2 \, R_e \tag{2}$$

Here, I_1^2 is the square rms value of the fundamental component.

Instantaneous power of the compensator is the difference between the total real instantaneous power required by the load and the instantaneous power supplied by the source.

$$p_C(t) = p_L(t) - p_S(t) \tag{3}$$

In this equation, the active power exchanged by the compensator has to be null, this is

$$P_C = \frac{1}{T}\int p_C(t)\,dt = 0 \tag{4}$$

When the average values are calculated in the equation (3), and when equations (2) is taken into account,

$$0 = \frac{1}{T}\int p_L(t)\,dt - I_1^2 \, R_e \tag{5}$$

Therefore, the equivalent resistance can be calculated by

$$R_e = \frac{\frac{1}{T}\int p_L(t)\,dt}{I_1^2} \tag{6}$$

Figure 1 shows the system with series active filter, parallel passive filter and non-sinusoidal load. The aim is that the compensation equipment and load have ideal behaviour from the PCC. According to the equation (1), the voltage at the connection point of the active filter can be calculated as follows

$$\boldsymbol{v}_{PCC} = \frac{P_L}{I_1^2}\,\boldsymbol{i} \tag{7}$$

Here, $\boldsymbol{i}$ is the source current vector and P_L is the load average power.

The reference signal for the output voltage of the active filter is

$$\boldsymbol{v}_C^* = \boldsymbol{v}_{PCC} - \boldsymbol{v}_L = \frac{P_L}{I_1^2}\,\boldsymbol{i} - \boldsymbol{v}_L \tag{8}$$

When the active filter supplies this compensation voltage, the set load and compensation equipment will behave as a resistor with a R_e.

3. Analysis Based on State Variables

Figure 1 shows the topology of analyzed hybrid filter. The passive power filter is formed by two LC branches tuned to the 5th and 7th harmonics. To represent the system by means of state variables, the circuit shown in figure 2 is used. It presents the equivalent single-phase model of the circuit shown in figure 1. This model will be used to analyze the system behaviour in the presence of harmonics different from the fundamental one.

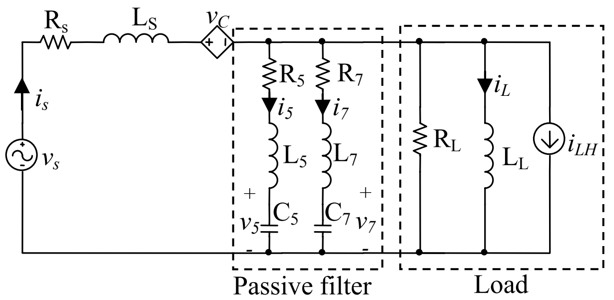

Fig 2. Single-phase model

The elements present in figure 2 are:

v_S: source voltage;
R_s, L_s: source resistor and inductance;
R_5, C_5, L_5, R_7, C_7, L_7: resistance, capacitance and inductance of the LC branches tuned at 5th and 7th harmonics;
R_L: resistor, which models the active power on the load;
L_L: inductor, which models reactive power on the load;
i_{LH}: load harmonics, at the fundamental harmonic is null.

When the active and passive filter are not connected, the system behaviour can be analyzed by the state equation

$$\begin{aligned} \dot{\boldsymbol{x}}' &= \boldsymbol{A}'\,\boldsymbol{x}' + \boldsymbol{B}'\,\boldsymbol{v} \\ \boldsymbol{y} &= \boldsymbol{C}'\,\boldsymbol{x}' + D'\,\boldsymbol{v} \end{aligned} \tag{9}$$

Here, the state vector is

$$\boldsymbol{x}' = \begin{bmatrix} i_S & i_L \end{bmatrix}^T \tag{10}$$

The system matrix is defined by means of

$$\boldsymbol{A}' = \begin{bmatrix} -\dfrac{R_S + R_L}{L_S} & \dfrac{1}{L_S} \\ -\dfrac{R_L}{L_L} & 0 \end{bmatrix} \tag{11}$$

The inputs vector is

$$\boldsymbol{v} = \begin{bmatrix} v_S & i_{LH} \end{bmatrix}^T \tag{12}$$

This is multiplied by matrix ***B'***

$$\boldsymbol{B'} = \begin{bmatrix} \frac{1}{L_S} & \frac{R_S}{L_S} \\ 0 & -\frac{R_L}{L_L} \end{bmatrix} \tag{13}$$

If the source current is chosen as the output variable, the matrix ***C'*** is

$$\boldsymbol{C'} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{14}$$

And the matrix ***D'*** = [0].
When the system is compensated, the state equation can be written as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\ \boldsymbol{x} + \boldsymbol{B_1}\ v_C + \boldsymbol{B_2}\ \boldsymbol{v} \\ \boldsymbol{y} &= \boldsymbol{C}\ \boldsymbol{x} + \boldsymbol{D_1}\ v_C + \boldsymbol{D_2}\ \boldsymbol{v} \end{aligned} \tag{15}$$

Where the state vector is

$$\boldsymbol{x} = \begin{bmatrix} i_S & i_5 & i_7 & i_L & v_5 & v_7 \end{bmatrix}^T \tag{16}$$

The A matrix is expressed as:

$$\boldsymbol{A} = \begin{bmatrix} -\frac{(R_S + R_L)}{L_S} & \frac{R_L}{L_S} & \frac{R_L}{L_S} & \frac{R_L}{L_S} & 0 & 0 \\ \frac{R_L}{L_5} & -\frac{(R_L + R_5)}{L_5} & \frac{R_L}{L_5} & \frac{R_L}{L_5} & -\frac{1}{L_5} & 0 \\ \frac{R_L}{L_7} & -\frac{R_L}{L_7} & -\frac{(R_L + R_7)}{L_5} & -\frac{R_L}{L_7} & 0 & -\frac{1}{L_7} \\ \frac{R_L}{L_L} & -\frac{R_L}{L_L} & -\frac{R_L}{L_L} & -\frac{R_L}{L_L} & 0 & 0 \\ 0 & \frac{1}{C_5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_7} & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

$\boldsymbol{B_1}$ is the vector

$$\boldsymbol{B_1} = \begin{bmatrix} -\frac{1}{L_S} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{18}$$

And the $\boldsymbol{B_2}$ vector is defined as follows:

$$\boldsymbol{B}_2 = \begin{bmatrix} \frac{R_L}{L_S} & -\frac{R_L}{L_5} & -\frac{R_L}{L_7} & -\frac{R_L}{L_L} & 0 & 0 \\ \frac{1}{L_S} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{19}$$

This multiplies the input vector, defined by

$$\boldsymbol{v} = \begin{bmatrix} v_S & i_{LH} \end{bmatrix}^T \tag{20}$$

When the source current is established as the output variable, the matrix ***C*** is

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

And finally, $\boldsymbol{D_1}$ = [0] and $\boldsymbol{D_2}$ = [0].

With the control strategy proposal in (1), at frequencies different from fundamental $i_S = 0$, therefore, v_C must be

$$v_C = i_5\ R_L + i_7\ R_L + i_L\ R_L + i_{LH}\ R_L + v_s \tag{22}$$

Its matrix form can be defined as

$$v_C = \boldsymbol{B_{11}}\ \boldsymbol{x} + \boldsymbol{B_{12}} \boldsymbol{v} \tag{23}$$

Here,

$$\boldsymbol{B_{11}} = \begin{bmatrix} 0 & R_L & R_L & R_L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

$$\boldsymbol{B}_{12} = \begin{bmatrix} R_L & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{25}$$

The state equation (8) can be rewrite as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \left(\boldsymbol{A} + \boldsymbol{B_1}\ \boldsymbol{B_{11}}\right)\boldsymbol{x} + \left(\boldsymbol{B_2} + \boldsymbol{B_1}\ \boldsymbol{B_{12}}\right)\ \boldsymbol{v} \\ \boldsymbol{y} &= \boldsymbol{C}\ \boldsymbol{x} + \boldsymbol{D_1}\ v_C + \boldsymbol{D_2}\ \boldsymbol{v} \end{aligned} \tag{26}$$

To corroborate the behaviour of the system, the figure 1 circuit has been considered with the element values indicate in table I.

TABLE I. Passive element values

L_s	R_s	L_5	R_5	C_5	L_7	R_7	C_7	L_L	R_L
2.8 mH	1.8 Ω	13.5 mH	2.1 Ω	30 μF	6.75 mH	1.1 Ω	30 μF	600 mH	13.7 Ω

The bode diagram of the state equations (9), (15) and (26) allows the analysis for different situations.
Figures 3 shows the gain of the system without compensating, with passive filter and with active filter respectively, when the source i_{Lh} is only taken in to account as input.
With the active filter connected, the gain is lower than without it, which demonstrates that the source current waveform improves at frequencies different of the fundamental.
On the other hand, when the passive filter is connected as compensation equipment alone, the gain is greater in approximately frequencies between 500 to 1400 rad/s (about 80-225 Hz), therefore, if the load generates the 3[rd] harmonics, when the passive filter is connected its rms value will increase.

With the active and passive filter connected, the gain is lower than without active filter, even for tuning frequencies of the passive filter.

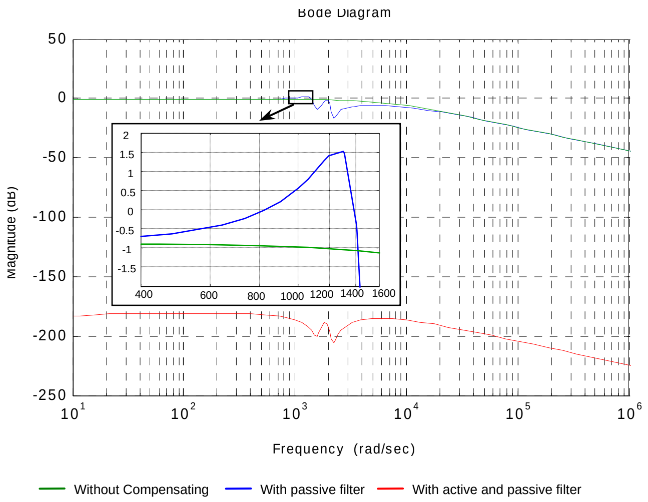

Fig. 3. Bode diagram, input i_{LH}

The state equation also let the analysis of poles and zeros and to study the system stability. The three situations present the poles in the left semiplane, therefore, the system is stable. Figure 4 shows the pole-zero diagram and table II the pole values.

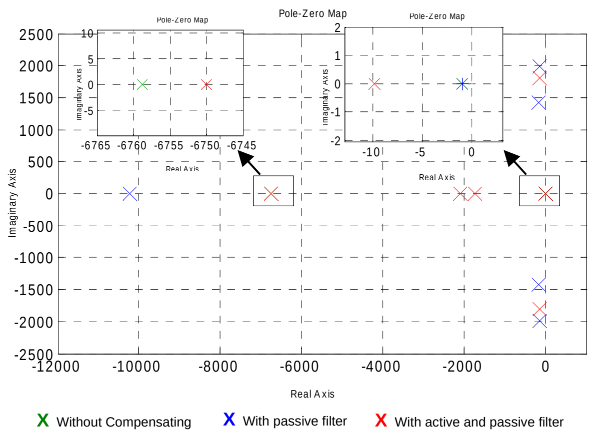

Fig. 4. Pole-zero map

TABLE II. Poles of the system

	Without compensating	With passive filter	With active and passive filter
Poles	s_1=-0.92 s_2=-6.76·10^3	s_1=-0.92 s_2=-1.02·10^4 s_3=-173+j1.42·10^3 s_4=-173-j1.42·10^3 s_5=-144+j1.98·10^3 s_6=-144-j1.98·10^3	s_1=-9.84 s_2=-6.75·10^3 s_3=-1.74·10^3 s_4=-2.11·10^3 s_5=-134+j1.81·10^3 s_6=-134-j1.81·10^3

The passive filter can end up being harmonics drains of other non-linear loads connected near the PCC and therefore overloads can happen, which is a drawback. This situation can be analyzed by means of Bode diagram when the v_S source is considered as input. Figure 5 shows the three situations.

With the passive filter, the gain is greater than without compensator for frequencies between 670 rad/s and 3100 rad/s approximately (between 100-500 Hz), therefore, at this frequencies, when the passive filter is connected the harmonics values are greater than without filter. It can be improved with the active filter since the Bode diagram presents a small gain.

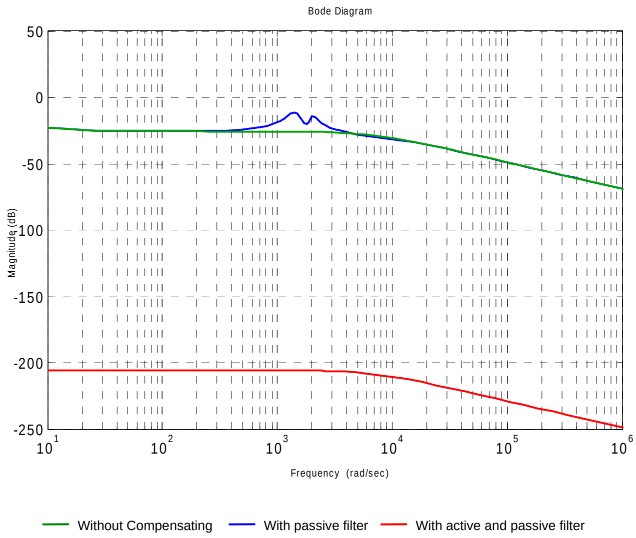

Fig. 5. Bode diagram, input v_s

4. Experimental results

The developed experimental prototype scheme is shown in figure 1. It is a three-phase system supplied by a sinusoidal balanced three-phase 100 V rms source. The converter is a Semikron SKM50GB123-type IGBT bridge on the DC side, where two 100 V capacitors are connected. On the AC side, an LC filter has been included to eliminate high switching frequency, with 13.5 mH inductance and 50 µF capacitance. This set is matched to the power system by means of three single-phase transformers with a turn ratio of 1:1 to ensure galvanic isolation.

The passive power filter is formed by two LC branches tuned to the 5th and 7th harmonics. The values of each passive element are included in table I.

The control strategy was implemented in a general application data acquisition & control cards compatible with Matlab-Simulink and developed by dSPACE.

The non-linear load consists of three single-phase uncontrolled rectifiers with a 55 mH inductor and a 12.5 Ω resistor connected in series on the DC side. A three-phase power quality meters, Fluke 434, was used to measure the THD, harmonics and powers. Table III summarizes some measures for the phase a.

Figures 6, 7 and 8 show the three source currents, before the compensation, when is only connected passive filter and when the active filter and passive filter is connected. These waveforms were obtained using a LECROY Wavesurfer 424- type oscilloscope.

With the passive filter the 5^{th} and 7^{th} harmonics values decrease, however the 3^{rd} harmonic value increases, consequently the source current THD goes up from 26.8 % to 31.1%. It is due to the increase of system gain when the passive filter is only connected. This fact was analyzed in the previous section.

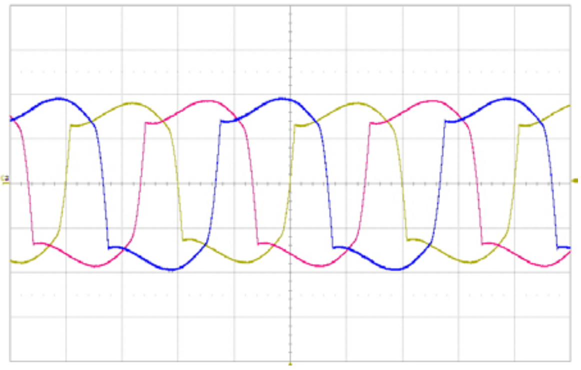

Fig. 6. Source current, system without compensating

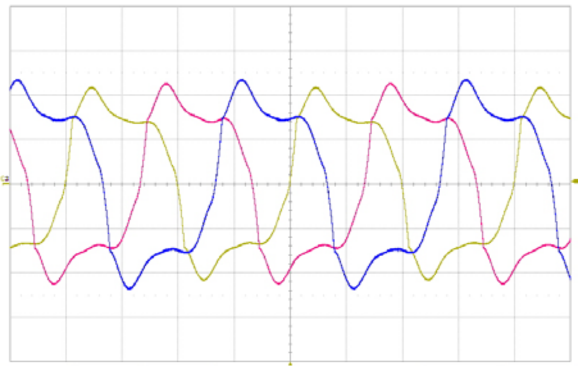

Fig. 7. Source current, system with passive filter

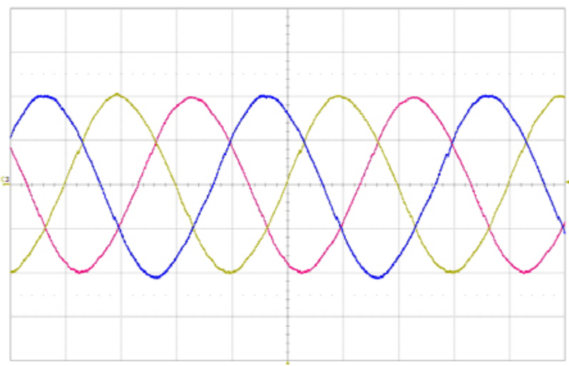

Fig. 8. Source current, system with active and passive filter

With the active filter connected, the current THD decreases until 2.7 %, the harmonics values are almost null. It is due to the reduction of the gain of the system. The power factor measured is unity, which is the aim for the fundamental frequency.

4. Conclusion

A control algorithm for a hybrid power filter constituted by a series active filter and a passive filter connected in parallel with the load is proposed. The control strategy is based on the dual vectorial theory of electric power.
System state model has been obtained and the system behaviour has been analyzed from the state equations for each situation. The analysis developed has allowed the knowledge of the system dynamic behaviour and the stability margins in each situation. This allowed an experimental prototype to be developed.
The new control approach achieves the following targets:
- The hybrid filter and load set are behaviour resistive. This fact eliminates the risk of overload due to the current harmonics of non-linear loads close to the compensated system.
- Series and/or parallel resonances with the rest of the system are avoided because compensation equipment and load are resistive behaviour.
- The active filter improves the harmonic compensation features of the passive filter and compensates the reactive power, achieving unit power factor.

Experimental and simulation results are presented. This allows verification of the developed theoretical analysis.

TABLE III. Experimental results, phase a

	Without compensating		With passive filter		With active and passive filter	
	V	I	V	I	V	I
THD %	8.7	26.8	7.4	31.5	1.3	2.7
RMS(A)	97.4	5.9	99.3	5.9	98.6	5.5
Fund.(A)	97	5.7	99	5.6	98.6	5.5
H3 (A)	4.2	1.2	5.7	1.7	0.5	0.1
H5 (A)	3.8	0.7	2	0.4	0.1	0
H7 (A)	3.4	0.5	0.8	0.1	0.1	0
H9 (A)	3	0.3	1.6	0.9	0.1	0
P(kW)	0.54		0.56		0.55	
Q(kvar)	0.17(i)		0.03(c)		0.00	
S(kVA)	0.56		0.56		0.55	
PF	0.91		0.94		1	

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