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State estimation for large power distribution systems

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Abstract. This article presents the modifications and improvements that have been included over the classic formulation of the state estimation to be able to run in real networks with large power distribution systems. Improvements include a new penalty term that keeps bus consumption or power generation within an allowed band and the inclusion of dynamic elements such as voltage regulators and capacitor batteries.

A complete algorithm is presented that is capable of operating on large power systems in near real time. This algorithm and the results have been tested with numerous cases of real distribution networks covering large geographical areas, obtaining very good results and demonstrating the enormous usefulness of this approach.

Key words

State estimation, measurement quality, large power distribution systems.

1. Introduction

The state estimation (SE) in power systems is a central piece for the supervision and quality improvement of these systems. SE receives field measurements from the remote units through a data acquisition system (SCADA). Using these measurements, the network inventory and the set of non-linear power equations, SE fine-tunes the state of each bus (the voltage module and angle) minimizing the weighted quadratic error, known as the Weighted Least Square (WLS) method.

Among the benefits of this approach are the ability to handle errors in measuring equipment and to discern bad measures, but it is necessary to expand or modify the original formulation [1][2] for use in real large networks.

Among the limitations of the approach are that they do not impose any restriction on the capacity to generate or consume of each bus (which is an unrealistic scenario for any distribution network) and that they do not contemplate dynamic elements such as batteries and voltage regulators whose behaviour depends on the value of the state in each iteration.

In addition, it describes how to efficiently manage large networks with more than 100000 buses in a few minutes from the initial state and in a few seconds from the previous solution, which enables the use of the algorithm for near real-time monitoring systems.

2. State estimation

The state estimator solves the well-known WLS optimization problem that tries to minimize the error of available measures: $e_i = z_i - h_i(x_1, x_2 \dots x_n)$.

Where z is the m-size vector of the available measurements and h is the vector of the nonlinear power equations dependent on the n-size state vector, with n being double minus one of the number of system buses.

Therefore, the following objective function is defined:

$$f = \sum_{i=1}^{m} (w_i(z_i - h_i(x_1, x_2 \dots x_{2\cdot n})))^2 = \sum_{i=1}^{m} (w_i e_i)^2$$

Where w is the weight vector of each measurement, which typically uses the inverse of the standard deviation of the measuring device as weight, i.e. $w_i = \frac{1}{\sigma_i}$
To find the minimum of the target function, partial derivatives are calculated and equalled to zero:

$$H^T \cdot W \cdot e = 0 \ (1)$$

Where H is the Jacobian matrix of partial derivatives of size (m,n): $H = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_m}{\partial x_1} & \cdots & \frac{\partial h_m}{\partial x_n} \end{bmatrix}$ and W is the diagonal matrix of weights of size (m,m) and *e* is the vector of errors of size m.

Equation 1 contains m nonlinear equations dependent on n variables that cannot be solved directly, which forces to use an iterative procedure such as Newton-Raphson which approximates nonlinear equations by their first term of the Taylor series, i.e.:

$$h_i(x_1, x_2 \dots x_n) \approx h_i(x_1{}^0, x_2{}^0 \dots x_n{}^0) + \sum_{j=1}^{n} \nabla x_j^0 \frac{\partial h_i}{\partial x_j}$$

With this approximation, the problem becomes linear at the cost of needing to iterate until converging to a solution close to the optimum. Substituting *h* in equation 1 gives:

$$H^T \cdot W \cdot H \cdot \nabla x = H^T \cdot W \cdot e \text{ (2)}$$

Where grouping $G = H^T \cdot W \cdot H$ and $b = H^T \cdot W \cdot e$, it is about solving the classic linear system $G \cdot x = b$ where G is usually called the gain matrix.

The following section will give more details of the method used to solve this system and avoid numerical problems.

In each iteration, the updated status vector is obtained and the new error *e* is calculated, stopping the iteration when an average error below a threshold is obtained [1].

3. Entering Restrictions

Equation 2 presents the basis of the state estimation algorithm. As can be seen, it does not apply any restriction to the active or reactive power that buses can consume or generate, which in practice generates mathematically correct but totally unrealistic solutions.

To do this, a WLS problem can be solved with restrictions for some states:

$$A \cdot x = b \text{ with } a_i < x_i < b_i$$

This type of equations can have convergence problems, and introduces the difficulty of associating the power restrictions in the buses with the value of the states.

The approach adopted by the authors is the introduction of an additional term in equation 2 that penalizes solutions that are outside the bands allowed for the desired buses. As will be demonstrated, it has excellent convergence properties and takes advantage of much of the calculations already made to form equation 2, so it barely introduces computational cost in its resolution.

All buses are classified into two categories, passive buses as those that cannot consume or generate, and active buses as those buses that can, for example, a transformation center. These centers have some nominal values of power and installed generation, which gives a band of allowed power.

Passive buses

This case is directly treated by adding a measure of P and Q equal to zero with a weight x times greater than the highest weight available.

It is also possible to formulate a WLS problem with equality restrictions to contemplate the values of P and Q equal to zero for these buses [4], although the authors have found the method of adding virtual measures, robust and simple, as long as no very high weights are used that can cause numerical problems.

Adding these fictitious measures not only ensures that these buses do not consume or generate, but also practically guarantees that the redundancy factor is greater than 1, thus reducing the degrees of freedom of the system.

Active buses

This case requires a modification of the approach shown in the previous section. The first step is to check whether there is a real measure of P and Q, so that, if there is no such measure, an estimated fictitious measure of P or Q whose value is the midpoint of the permitted band and whose weight is fixed and x times lower than the equivalent real measures is added.

This step has a double purpose, on the one hand, it guides the estimator to obtain values within the band and on the other hand, it assures that all the active buses have measurement (either real or fictitious).

In this way, the error or penalty of a measure *i* of P or Q that is out of range is defined as:

$$ep_i = \begin{cases} -h_i - C_i \ si \ -h_i > C_i \\ h_i - G_i \ si \ h_i > G_i \\ 0 \ in \ any \ other \ case \end{cases}$$

Where C_i is the contracted or installed power value allowed for that bus and G_i is the generated power allowed for that bus.

By defining the same objective function for this new term, and applying the same criterion of minimizing error, the additional penalty term is obtained:

$$H'^T \cdot W_p \cdot H' \cdot \nabla x = H'^T \cdot W_p \cdot e_p$$

Where H' is the same Jacobian matrix already calculated, but with the rows to zero where the bus is not active. W_p is the diagonal weight matrix with zero values where the bus is not an active bus and with a fixed weight (x times the error of the most precise equipment) for all P and Q measurements of active buses whose estimated value is out of range, i.e. $ep_i > 0$.

Note that H' is always multiplied by W_p so it is not necessary to obtain it, the matrix H already obtained can be used. This formulation has the advantage of being equivalent to the one shown in equation 2, which allows to reuse most of the calculations already made and minimizes convergence problems.

Adding all the terms together gives the equation of the state estimation with restrictions:

$$H^T \cdot (W + W_p) \cdot H \cdot \nabla x = H^T \cdot (W \cdot e + W_p \cdot e_p) \quad (3)$$

As indicated in [2], the *G* matrix may present numerical problems of precision and rounding that may affect the convergence of the system, or in the best of cases, a greater number of iterations necessary for its resolution.

Therefore, the authors recommend using the orthogonal factorization method that avoids calculating the G matrix and significantly reduces numerical problems:

$$\widetilde{H} = Q \cdot R \text{ where } \widetilde{H} = H \cdot (W + W_p)^{1/2}$$

Substituting in equation 3, it is obtained:

$$\widetilde{H}^T \cdot \widetilde{H} \cdot \nabla x = \widetilde{H}^T \cdot (W + W_p)^{-1/2} \cdot (W \cdot e + W_p \cdot e_p)$$

And considering that Q is orthogonal, $Q^T = Q^{-1}$, it is obtained:

$$R \cdot \nabla x = Q^T \cdot (W + W_p)^{-1/2} \cdot (W \cdot e + W_p \cdot e_p) \quad (4)$$

Equation 4 that can be solved by direct substitution since R is an upper triangular matrix. This resolution method has been superior in all aspects to solving the G matrix by decomposition, avoiding possible numerical problems and needing fewer iterations to converge.

4. Entering Dynamic Elements

A dynamic element is defined as a component whose behaviour depends on the states, i.e. module or voltage angle.

These components present the problem of introducing changes to the values already calculated in each iteration, such as changes in the Y-bus matrix. For this reason, they must be incorporated considering at all times that they have no impact on the efficiency of the algorithm so that it remains valid for large systems.

Next, two dynamic elements are detailed, the voltage regulators and the capacitor batteries, where the first one implies changes on the Y-bus matrix, while the second one in the vector of *z* measures.

Voltage regulators

The voltage regulator is modelled as a transformer whose tapping changes in order to try to maintain a fixed setpoint voltage in the secondary. The unit diagram of an ideal transformer is shown below:

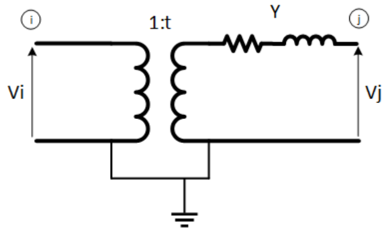

Fig. 1. Ideal transformer

To include this model, the Y-bus matrix between buses *i* and *j* is modified as follows: $\begin{bmatrix} |t|^2 Y & -t^* Y \\ -tY & Y \end{bmatrix}$

The following algorithm is used to efficiently include the voltage regulator within the state estimation at the start of each iteration:

- The new tap of iteration k is calculated: $t_k = \frac{V_c}{V_i}$, where V_c is the setpoint voltage.
- The new tap is compared with the previous one: $|t_k - t_{k-1}| > \varepsilon$. In this case, real t is assumed, i.e. it does not apply phase changes.
- If the above condition is met, the Y-bus matrix is updated as follows:

$$Y_{ii}{}^{new} = [Y_{ii}] + (-t_{k-1}{}^2 + t_k{}^2) \cdot Y$$
$$Y_{ij} = Y_{ji} = -t_k \cdot Y$$

Note the incremental change on the Y_{ii} component of the Y-bus matrix, which implies a low computational cost and in successive iterations the tap increase is smaller until it falls below the established epsilon, which guarantees good convergence properties.

Capacitor Batteries

The capacitor battery is an element capable of injecting reactive to the bus to which it is connected with the following ratio:

$$Q_i = S_n \left(\frac{V_i}{V_n}\right)^2$$

Where S_n and V_n are the nominal power and voltage of the equipment.

To include this dynamic element, whose reactive injected depends on the voltage module of the bus to which it is connected, a virtual Q measurement is included with an accuracy equivalent to that of power equipment.

The following algorithm is executed at the beginning of each iteration:

- The new measure of iteration k is calculated: $Q_i^k = S_n \left(\frac{V_i}{V_n}\right)^2$.
- The new measure is compared with the previous one: $|Q_i^k - Q_i^{k-1}| > \varepsilon$.
- If the above condition is met, the measurement value is updated in the *z* vector.

In each iteration, the Q increment will be lower until it is below the established epsilon, which facilitates the convergence of the algorithm.

5. Considerations for large systems

A key factor for the usefulness of a state estimation is to be able to execute it on real large systems, with hundreds of thousands of buses that distribute power to large geographical areas.

In order to do this, several factors that have a direct impact on the efficiency of the algorithm must be taken into consideration.

Generation of bags

First, the number of bags in the study region is calculated, where a bag is defined as an area that do not have electrical connections with other areas.

In this case, the simplification of assuming that the high voltage substation node is a slack node is introduced, so there is no interconnection upstream of the substations.

These bags do not have any Y-bus matrix element that connects them, so two bags are defined as disconnected if they meet condition $Y_{i,j} = 0,$ with $i \neq j$ for all buses i of bag A and for all buses j of bag B.

A state estimation is executed for each bag in different threads in parallel, taking advantage of the multi-threading power of the current processors, obtaining a double benefit, on the one hand, smaller problems are solved, with a smaller number of buses and on the other hand they are executed in parallel, significantly reducing the necessary calculation time.

Once a bag has been resolved, the state vector of the bag is stored in memory so that successive executions are carried out in near real time, and can be used for monitoring and system operation.

When a maneuver is performed in one of the bags, it is necessary to discard the previous state and run the algorithm again, since the Y-bus matrix itself has changed.

Bus reduction

To increase performance, it is recommended to reduce the Y-bus matrix which, without loss of precision or information, reduces the dimensions of the matrix.

To do this, all nodes that meet the following conditions are iterated:

- That does not contain an active element, such as a center, a battery, etc.
- That is not a bar.
- That has less than three segments connected.
- That does not have a measurement associated, either directly to the bus or any of the segments it connects.

On all the nodes that fulfill the previous conditions, a Kron reduction is made to eliminate this bus from the matrix, where the new values of the Y-bus matrix are: $Y_{jk\,(new)} = Y_{jk} - \frac{Y_{jp} \cdot Y_{pk}}{Y_{pp}}$, being p the bus to eliminate.

The new reduced Y-bus matrix is obtained, deleting all the rows and columns of the eliminated p-buses. This matrix, which is electrically identical to the original, and where no information has been lost, has much smaller dimensions than the original and, therefore, has a very positive impact on calculation times.

Sparse Algebra

The Y-bus matrix directly reflects the connections between the buses, therefore, except for their diagonal, most values will be zero. This is true even in grid networks, since most buses are connected to other buses by a few segments.

To take advantage of this fact, sparse algebra must be applied to the mathematical operations that are performed. This concept should be extended to all calculations, not only to the manipulation of matrices, e.g. power equations, Kron reduction, etc., it should only be iterated by elements that are different from zero rather than by the whole row or column of the matrix.

The only point where it is possible that an improvement is not obtained is in the resolution of the system $G \cdot x = b$, since in general the gain matrix is not as dispersed as the Y-bus or the Jacobian matrix, for that reason, the authors solve equation 4 with dense matrices.

Using sparse algebra correctly is one of the improvements of greater impact that can be applied to the state estimation, being essential its use for real large systems with hundreds of thousands of buses.

Stop condition

The stop condition described in the second section [1] is too simple and not very useful for large real systems, with restrictions and dynamic elements such as those described in this article.

For this purpose, the authors present a specific stop algorithm with better precision and convergence properties:

1. The step parameter $p = 1$ is set.
2. The increment of the state vector ∇x_i is obtained according to equation 3 for iteration i.
3. Calculate the new state vector $x_{i+1} = x_i + \text{p} \cdot \nabla x$.
4. The absolute mean value of all the errors of the not eliminated measurements (MAE) is calculated.
5. If $MAE_i < MAE_{i-1}$ goes back to point 1 and continues to iterate.
6. If $p > \varepsilon$, reduce the step to half $p^{new} = {}^{p}/_{2}$, and go back to point 3.
7. Otherwise, the best possible estimate is the one obtained by x_i, and the execution is finished.

This algorithm has the virtue of not fixing an objective error, since it allows iterate while it fulfills the condition of improving the previous result, besides allowing to shorten the step when it is needed, improving notably its properties of convergence.

Bad data

One of the virtues of the state estimator is its ability to detect and correct possible bad measures. In [1][2][3] the detection of anomalous measures is described based on the fact that the weighted sum of Gaussian errors squared follows a *chi-square* distribution with $k = m - n$ degrees of freedom.

In this way, it is possible to check at a confidence level α, if the condition is satisfied that there are no anomalous measures according to the equation: $\Pr\left(f < X_{k,\alpha}^{2}\right) = 1 - \alpha$

This approach, while correct, has several drawbacks:

- It is necessary to know the actual (and not estimated) standard deviation of all field measuring devices.
- It assumes that the only source of errors comes from measuring devices. This assumption is too simplistic, since in any real SCADA, there are many other possible sources of errors or inaccuracies, such as inverted measurements, frozen data, time delays, etc.

All this leads to the proposal of an alternative method for detecting anomalous measures, which is simpler and more direct. Therefore, the standard error is calculated for all real measurements at the end of the iterations: $e_i = \frac{|z_i - h_i|}{\sigma_i}$.

The largest calculated error of the vector *e* is obtained, and if this error is greater than a fixed value, such as 3 (whose meaning is that it exceeds three times the standard deviation of the measuring device), the measurement is eliminated and iterated again. Each time, a single measurement is eliminated, since one bad measurement has the ability to impact the entire state vector.

As a summary of all that has been explained in this section, the following flowchart is shown:

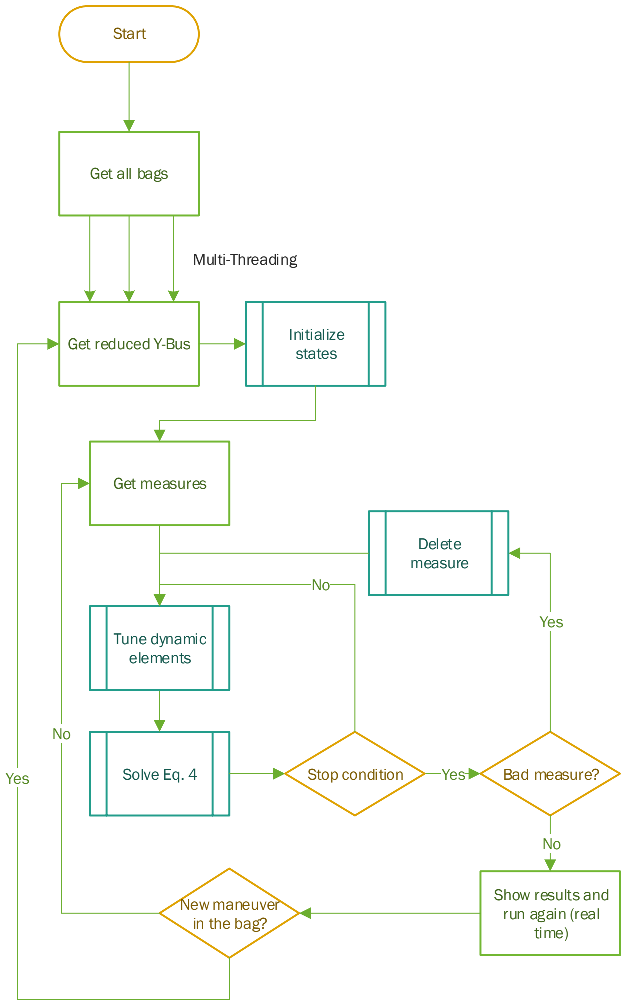

Fig. 2. State estimation flowchart

6. Case study

The results of a case study corresponding to a region of eastern Spain are presented. For reasons of confidentiality, centres and substations names have been masked.

The study network contains the following elements:

- 169147 buses.
- 44217 measures.
- 19100 centers.
- 106 power stations.

The execution of the algorithm shown in figure 2 obtains the following results:

- 88 independent bags.
- 321 bad measures detected (0.7%).
- 16.75 iterations on average per bag.
- A mean absolute error of 0.135.
- Total time of execution of 2.7 minutes.

The execution times of each bag once executed the first time, are less than a second, which allows monitoring of the network almost in real time.

To be able to inspect the results, a network representation of the reduced Y-bus matrix was used, where an edge is represented for each non-zero element that does not belong to the diagonal.

This type of representation has been very useful to explore and validate the results obtained by the state estimator.

The following figure shows the representation of a portion of the case study:

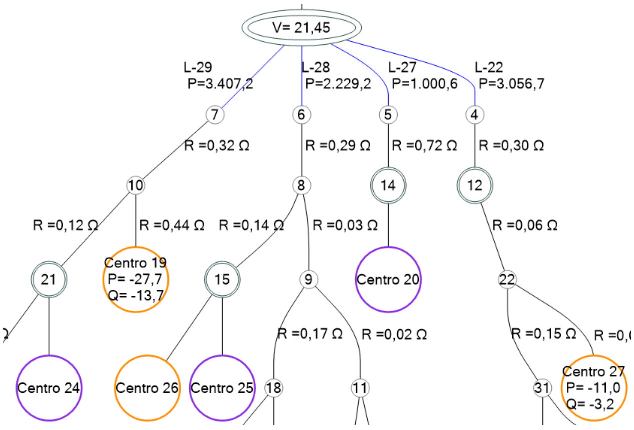

Fig. 3. Graph representation of the reduced Y-Bus matrix

7. Conclusions

A novel algorithm has been presented to execute a state estimation in large power distribution systems with hundreds of thousands of buses, capable of incorporating power and generation restrictions in the buses and naturally includes dynamic elements such as voltage regulators and capacitor batteries.

The solutions obtained by the state estimation with restrictions are much more faithful to reality as it forces to seek those solutions allowed by the configuration of elements capable of consuming or generating power, such as distribution centers, distributed generators, etc.

Likewise, the algorithm has been designed to work with large systems with hundreds of thousands of buses with the intention of being able to run in near real time to be incorporated into existing monitoring and management systems as another source of information.

The algorithm and the results have been tested with numerous cases of real distribution networks covering large geographical areas, obtaining very good results and demonstrating the enormous usefulness of these tools for the detection of anomalous measures and therefore for the improvement of the quality of information throughout the system.

There is an increasing trend towards scenarios in which a large number of redundant measures are available, and it is essential to be able to deal with this amount of information and deal correctly with possible measurement errors. This scenario will only increase, where redundant measures are already available even in low-voltage networks, so it will be essential to adopt similar techniques at all levels of energy distribution.

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