

Apparent power and power factor in unbalanced and distorted systems. Applications in three phase load compensations

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Abstract. The apparent power and the derived power factor are two of the quantities of the biggest use and application inside the Electric Engineering. However, nowadays it continues staying the controversy on their definitions and more appropriate meaning in the most general situations in unbalance and distortion, and unequal resistances in the distribution lines. In the last years they have been distinguished two focuses concerning the functional definitions of apparent power: the European approach, more theoretical and better developed, and the American approach, more practical focus but with smaller rigor. Although from the point of view of their practical application, the use of a definition or another doesn't suppose important numeric differences, if present differ from the conceptual point of view. This is made notice in applications that suppose the use of modern equipments of static compensation. The apparent power of the European approach in its conception only permits parallel compensation, while the apparent power of the American approach admits the series-parallel compensation. This paper introduces the definitions of apparent power of both approaches and it establishes a discussion on its application in the three-phases loads compensation.

Key words

Electric power, harmonics, unbalance, apparent power, power factor, compensation.

1. Introduction

The apparent power is one of the concepts of more importance and application inside the power engineering. It also constitutes the base of the definition of power factor that nowadays continues being a key merit figure for the measure of the energy efficiency of an electric consumption. Their definition for three-phase systems under the most general conditions, asymmetry and distortion, has raised a great controversy along the years, [1].

Recently, the pattern that has prevailed when treating the analysis to the three-phase systems with neutral conductor, is to consider it as a system of four-conductor according to an original idea introduced by Depenbrock , [1-2].

From here they join two different positions. The European approach that has their origin in Depenbrock, and the American approach sponsored by IEEE that leadership Emanuel. The European approach has been very well based from the theoretical point of view in the

last years thanks to the works of the own Depenbrock, [2] and Mayordomo et al., [3], and the subsequent works of Willems, [4-6] that have helped to clarify the situation. On the other hand, the IEEE approach has sought to introduce in the definition of apparent power a practical effective voltage, associated in a principle to the non-load losses, [7-10]. It is an interesting idea, but that it has not been completely resolved until the present time.

In fact, this last one has gone readapting in a continuous way the concept of apparent power from the original focus of Depenbrock until their proposal of the IEEE Std 1459-2000, [7], in which introduces an equivalent voltage different from the equivalent voltage of the European approach, [8]. Even after being published the IEEE Standard, the same Emanuel redefines the equivalent voltage concept and his interpretation, [6], [10].

In this communication the definitions of apparent power and power factor are analyzed from both focus types referred to the compensation process. It discusses their more appropriate use in a parallel compensation process or in a series-parallel compensation. Finally, an appropriate example of application is presented.

2. Apparent power

Two approaches for the definition of apparent power are using at the present time. The European approach that gives place to a power factor definition more appropriate to characterize the process of load compensation by means of a shunt compensator. And the American approach that allows to define a more appropriate power factor to characterize the compensation by means of a combined equipment of series-parallel connection of every time bigger use, fig 1.

The configuration of the system here considered whoever load is that supplies through a four conductor system each one of which it is supposed with a certain resistance value.

The definition of apparent power of more acceptances for the specialists of the topic in the last years is the interpretation of Buchholz. This way, the apparent power is the maximum active power that can be transmitted to the load for a given voltage waveform and some losses given in the feeding line. The voltage waveform is imposed by the network and the power factor defined as

the quotient between the active power and the apparent power, it relates the minimum losses with the real losses in the transmission, [5].

In view of the previously suitable definition, their determination becomes a process of maximizing of the power transmitted to the load subject to the condition of constant line losses. In that way the characteristic value, equivalent current appears, I_e , as the rms value of a three-phase current of positive sequence that produces the same losses that the actual current. The power apparent resultant of solving the outlined problem of ends is

$$S = 3V_e I_e \quad (1)$$

where V_e it is the three-phase voltage rms value of positive sequence that produces the same rms value that the voltage waveform imposed by the utility. Nevertheless, this voltage value comes determined in function of a voltage reference that it depends on the losses of the distribution line. One of the objectives of this work is to clarify in an analytic way this situation.

A. General analysis.

In this section it will be carried out a general analysis, for what will be considered that the conductor resistances are different, [4], [10]. This way, for a base resistance given, r , is defined the rates,

$$\rho_a = \frac{r_a}{r} \quad \rho_b = \frac{r_b}{r} \quad \rho_c = \frac{r_c}{r} \quad \rho_n = \frac{r_n}{r} \quad (2)$$

These rates in general will be different some of other, although later on it will be particularized to the case of, $\rho_a = \rho_b = \rho_c = 1 \neq \rho_n$.

The incoming active power to the load comes given for,

$$P = \Re \{ V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^* \} = \Re \{ V_a I_a^* + V_b I_b^* + V_c I_c^* + V_n I_n^* \} \quad (3)$$

where the terminal voltages of the load, included the neutral conductor, they are referred to an arbitrary reference anyone.

The determination process of the apparent power consists on maximizing the active power P , according to two constraints:

- The losses in the distribution lines, ΔP given for (4), they stay constant

$$\Delta P = r(\rho_a I_a^2 + \rho_b I_b^2 + \rho_c I_c^2 + \rho_n I_n^2) \quad (4)$$

- The current Kirchhoff law

$$I_a + I_b + I_c + I_n = 0 \quad (5)$$

Lagrange multiplier techniques permit to solve the problem.

The apparent power S , maximum active power that can be transmitted to the load for a given waveform of voltage and a given rms value of current (or lines losses), it is,

$$S = P_{\max} = 3V_e I_e \quad (6)$$

where the effective voltage and current are

$$V_e = \sqrt{\frac{1}{3} \sum_{\forall k} \frac{|V_k - V_{ref}|^2}{\rho_k}} \quad k = a, b, c, n \quad (7)$$

$$I_e = \sqrt{\frac{1}{3} \sum_{\forall k} \rho_k I_k^2} \quad k = a, b, c, n$$

Before continuing ahead, it is suitable to carry out two observations to the expression obtained for the apparent power S . The first one is that the effective current has a clear interpretation regarding the losses of the line. Indeed, the equivalent current I_e represents the rms value of a three-phases current of positive sequence that produces the same losses that the actual current, that is of (4),

$$\Delta P = r(\rho_a I_a^2 + \rho_b I_b^2 + \rho_c I_c^2 + \rho_n I_n^2) = r(\sqrt{3} I_e)^2 \quad (8)$$

The second remark refers to the voltage; the effective voltage is determined in function of the voltage rms values of the terminals concerning a voltage reference with a phasor given by

$$V_{ref} = \frac{\sum_{\forall k} V_k \rho_k}{\sum_{\forall k} \rho_k} \quad (9)$$

This voltage corresponds to the neutral point of a star built by four different branches, $\rho_a r$, $\rho_b r$, $\rho_c r$, $\rho_n r$.

The followed process up to now can be formalized from the mathematical point of view, considering the two factors that intervene in the expression of S like norms of fictitious current and voltage vectors, [3]. This is, it is defined the fictitious vector of current,

$$I = [\sqrt{\rho_a} I_a \sqrt{\rho_b} I_b \sqrt{\rho_c} I_c \sqrt{\rho_n} I_n]^T \quad (10)$$

and a fictitious vector of voltage

$$V = \left[\frac{V_a - V_{ref}}{\sqrt{\rho_a}} \quad \frac{V_b - V_{ref}}{\sqrt{\rho_b}} \quad \frac{V_c - V_{ref}}{\sqrt{\rho_c}} \quad \frac{V_n - V_{ref}}{\sqrt{\rho_n}} \right]^T \quad (11)$$

This allows establishing the definition of S like the product of the vector norms (10) and (11),

$$S = \|V\| \|I\| \quad (12)$$

and it remains invariant the active power P,

$$P = \Re\{V^T I^*\} \quad (13)$$

The maximizing process of (3) subjected to the constraints (4) and (5) according to the Lagrange multipliers method, it determines the currents that produce the maximum power (6),

$$I_{ka} = \frac{P}{\sum_{\forall k} \frac{|V_k - V_{ref}|^2}{\rho_k}} \frac{V_k - V_{ref}}{\rho_k} \quad (14)$$

$k = a, b, c, n$

These represent the currents, that circulates for a conductance networks G/ρ_k different for each branch, with a rms value such that together with the voltage sets $V_k - V_{ref}$ gives the total active power of the load. Thus, one has a fictitious vector of active current,

$$I_a = \left[\sqrt{\rho_a} I_{aa} \sqrt{\rho_b} I_{ab} \sqrt{\rho_c} I_{ac} \sqrt{\rho_n} I_{an} \right]^T \quad (15)$$

whose components are related with the fictitious voltage vector in the form,

$$\sqrt{\rho_k} I_{ka} = G \frac{V_k - V_{ref}}{\sqrt{\rho_k}} \quad k = a, b, c, n \quad (16)$$

G is therefore the active part that can separate in the fictitious equivalent circuit of the load. On the other hand, the rms value of the current I_a is the smallest value in I and it transfers the power P that will produce the minimum losses in the line.

According to (14), (15) y (16) following the relationships,

$$\|I_a\| = G \|V\| \quad (17)$$

and

$$P = G \|V\|^2 = \|V\| \|I_a\| \quad (18)$$

These last relationships allow finding a meaning for the defined power factor as the quotient between the active power and the apparent power. Indeed, the relationship among the minimum losses of power in the line and the actual losses are,

$$\frac{\Delta P_{\min}}{\Delta P} = \frac{r \|I_a\|^2}{r \|I\|^2} \quad (19)$$

since $\|I_a\|$ is the minimum value of $\|I\|$ that transports the power P. From (17) and (18),

$$\frac{\Delta P_{\min}}{\Delta P} = \left(\frac{P}{S} \right)^2 = (\text{PF})^2 \quad (20)$$

The definition of apparent power (12) it is the only one compatible with the physical meaning expressed for (20).

B. A particular case

In this section it will be considered the most habitual practical case where the conductor lines present the same resistance and the neutral conductor presents a resistance of different value, [5]. Let be, therefore, a three-phase four wire system in the most general conditions in voltage and current. It will be supposed that each line conductor has a resistance r and the neutral conductor has a resistance $r_n = \rho r$. Thus, the losses of the line come given by the relationship,

$$\Delta P = r (I_a^2 + I_b^2 + I_c^2 + \rho I_n^2) = r (\sqrt{3} I_e)^2 \quad (21)$$

The current fictitious vector is now,

$$I = \left[I_a \ I_b \ I_c \ \sqrt{\rho} I_n \right]^T \quad (22)$$

It is constituted by the rms values of the phases current and the rms value of the neutral current weighted by a factor. This factor is the square root of the rate of the neutral resistance to the line resistance. On the other hand, the voltage fictitious vector adopts the form,

$$V = \left[V_a - V_{ref} \ V_b - V_{ref} \ V_c - V_{ref} \ \frac{V_n - V_{ref}}{\sqrt{\rho}} \right]^T \quad (23)$$

The reference voltage is,

$$V_{ref} = \frac{V_a + V_b + V_c + \frac{V_n}{\rho}}{3 + \frac{1}{\rho}} \quad (24)$$

Thus, each terminal voltage of the load is measured with regard the virtual star point of four voltmeter resistances ($r, r, r, \rho r$) connected in parallel with the load. If the conductor line resistances are the same ($\rho=1$), then the reference coincide with the neutral of a virtual symmetrical star. If it was considered negligible the neutral resistance ($\rho=0$), then the reference coincide with the neutral terminal.

The apparent power is determined for (12),

$$S = \|V\| \|I\| = 3 V_e I_e \quad (25)$$

where now the effective voltage and current values take the form:

- For the voltage,

$$V_e = \frac{1}{\sqrt{3}} \frac{\sqrt{|V_a - V_{ref}|^2 + |V_b - V_{ref}|^2 + \dots + |V_c - V_{ref}|^2 + \frac{1}{\rho} |V_n - V_{ref}|^2}}{\sqrt{3 + \frac{1}{\rho}}} \quad (26)$$

or in function of the phase to phase voltage and voltages referred to the neutral one,

$$V_e = \frac{1}{\sqrt{3}} \frac{1}{\sqrt{3 + \frac{1}{\rho}}} \frac{\sqrt{|V_{ab}|^2 + |V_{bc}|^2 + |V_{ca}|^2 + \dots + \frac{|V_{an}|^2 + |V_{bn}|^2 + |V_{cn}|^2}{\rho}}}{\rho} \quad (27)$$

- For the current,

$$I_e = \sqrt{\frac{1}{3} (I_a^2 + I_b^2 + I_c^2 + \rho I_n^2)} \quad (28)$$

In many occasions they are useful the expressions of the effective voltage and current in function of the sequence components,

- For the voltage,

$$V_e = \sqrt{V_+^2 + V_-^2 + \frac{1}{1+3\rho} V_0^2} \quad (29)$$

- For the current,

$$I_e = \sqrt{I_+^2 + I_-^2 + (1+3\rho) I_0^2} \quad (30)$$

where the subscripts +, -, and 0 correspond to the positive, negative and zero sequences, respectively.

3. The IEEE approach

The American approach establishes the same operative definition that the European approach, this is the definition of S, but with a different definition and interpretation for the V_e . This way, for the IEEE Std 1459-2000, [7], [9], the apparent power is the maximum active power that can be transmitted by a balanced system that produces the same impact of voltage and the same impact of current in the network. Here impact refers to what isolation it is necessary and what losses in no-load they are expected, and what losses it takes place in the lines the current magnitude. The last shading on this focus appears in [9]. There, that definition of defined equivalent voltage starting from the losses that depend on the voltage gives way in favour of the loads active powers approach, [6]. This is, it is defined the ratio,

$$\xi = \frac{P_\Delta}{P_Y} \quad (31)$$

where P_Δ is the power absorbed by the floating Y and the Δ -connected loads, and P_Y is the active power absorbed by the Y-connected loads. The equivalent voltage for this approach doesn't depend on the line resistances and it comes determined in function of ξ .

Indeed, in the determination of the equivalent voltage V_e , it is supposed that the load consists of a resistance sets connected in Y and a remaining group connected in Δ . Each group characterized by an equivalent resistance R_Y and R_Δ , respectively. The equivalence approach is based on identical electrothermal effects, that is,

$$\begin{aligned} \frac{V_{an}^2 + V_{bn}^2 + V_{cn}^2}{R_Y} + \frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{R_\Delta} &= \\ &= \frac{3V_e^2}{R_Y} + \frac{9V_e^2}{R_\Delta} \end{aligned} \quad (32)$$

The rate of powers is,

$$\xi = \frac{P_\Delta}{P_Y} = \frac{\frac{9V_e^2}{R_\Delta}}{\frac{3V_e^2}{R_Y}} = \frac{3R_Y}{R_\Delta} \quad (33)$$

and it is substituted (33) in the expression (32), it is (34),

$$\begin{aligned} \frac{V_{an}^2 + V_{bn}^2 + V_{cn}^2}{R_Y} + \frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{3R_Y} \xi &= \\ &= \frac{3V_e^2}{R_Y} + \frac{9V_e^2}{3R_Y} \xi \end{aligned} \quad (34)$$

that it place finally,

$$V_e = \sqrt{\frac{3(V_{an}^2 + V_{bn}^2 + V_{cn}^2) + \xi(V_{ab}^2 + V_{bc}^2 + V_{ca}^2)}{9(1+\xi)}} \quad (35)$$

It is also possible to prepare the expression for the effective voltage starting from the sequence components,

$$V_e = \sqrt{V_+^2 + V_-^2 + \frac{1}{1+\xi} V_0^2} \quad (36)$$

For the effective current the IEEE approach uses the same definition that (28) or (30).

The electrical meaning associated to these concepts is summarized in the following one enunciated; the system that allows the maximum power transfer is one perfectly balanced and symmetrical with a line current I_e and a phase to neutral voltage V_e .

The IEEE Std. 1459 use $\rho_n = 1$, since it is a parameter difficult to know. Emanuel, [10], responsible of the Standard one, suggests that when it can be measured or to be stimulated, let be this the value that is used. With regard

ζ , this is usually more difficult of knowing; the Std take the value 1. When $\zeta=3\rho$, the expressions of both approaches, the European and the American, they coincide.

When it has the analytic expressions of the equivalent voltages of both approaches in function of the sequence components, (36) y (29), it is recognized that they only differ in the contribution of sequence zero voltage. This is, when the voltage of sequence zero is null, both definitions coincide. Anyway for practical values of V_0 the difference among the two definitions is inside the error margin of the instrumentation measurements.

4. Shunt compensation versus series-shunt compensation

The definition of apparent power in the European approach considers the voltage imposed by the utility grid. It supposes, therefore, that is only possible to make load compensation by means of an equipment of shunt connection that gets that the fictitious currents are in phase with the fictitious voltages. This means that the load more compensator will have a unit power factor when it is equivalent to a resistive network that will be unbalanced in general. The IEEE approach considers that one obtains unit power factor when so much the voltage sets as the current sets conforms a balanced sinusoidal system of positive sequence phases. This requires making a shunt compensation of currents like a series compensation of voltages so much. At the present time it has series and parallel static compensator; series-parallel hybrid systems have even been researched constituting authentic universal equipment of compensation, [11]. The figure 1 presents a habitual configuration. Certainly this new type of universal compensator requires finding an appropriate definition of apparent power (and factor of power) that contemplates the double action it has in the currents and voltages.

In following these concepts they will be applied to a concrete practical example. Is it considered a load anyone that absorbs an active power P and that it is supplied through a line with phase conductor resistances r and a neutral conductor resistance $r_n = \rho r$, where ρ it represents the fraction of the neutral conductor resistance related to the phase conductor resistance. It will be supposed that applied three-phase voltage in the load terminals only has component of zero sequence, it is an extreme situation to simplify the numeric calculations in this example.

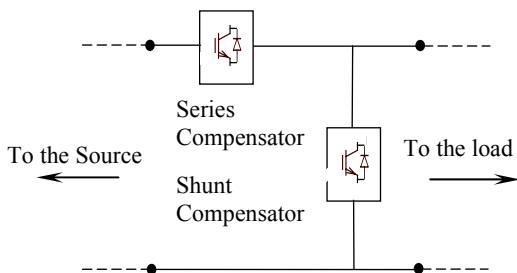


Fig. 1. Series-shunt compensation equipment

An equipment of active compensation is connected (APLC) in parallel with the load. After compensation the set load + compensator will behave as a group of three resistances of same value R , figure 1.

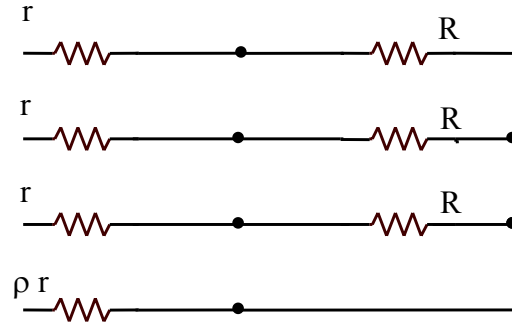


Fig 2. A balanced load supplied by a line with phase conductors of the same resistance and neutral conductor resistance of different value.

The power absorbed by the load is

$$P=3\frac{V_0^2}{R} \quad (37)$$

According to the European approach, the equivalent voltage is

$$V_e = \sqrt{\frac{V_0^2}{1+3\rho}} \quad (38)$$

and the equivalent current

$$I_e = \sqrt{(1+3\rho)I_0^2} \quad (39)$$

Then the apparent power is

$$S=3V_e I_e = 3V_0 I_0 = 3\frac{V_0^2}{R} \quad (40)$$

and the power factor is the unity, $PF=1$.

The apparent power indicates the maximum power that can absorb the original load for the losses in the line,

$$P_{loss} = r(\sqrt{3}I_e)^2 = 12rI_0^2 \quad \text{for } \rho=1 \quad (41)$$

A unit power factor means that it is not possible to improve the energy efficiency by means of shunt compensation.

According to the American approach, the equivalent voltage is,

$$V_e = \sqrt{\frac{V_0^2}{1+\xi}} \quad (42)$$

and the equivalent current coincides with the previous I_e . The apparent power is therefore,

$$S_e = 3V_e I_e = 3 \sqrt{\frac{1+3\rho}{1+\xi}} V_0 I_0 = 3 \sqrt{\frac{1+3\rho}{1+\xi}} \frac{V_o^2}{R} \quad (43)$$

In this occasion, the power factor is,

$$PF_e = \sqrt{\frac{1+\xi}{1+3\rho}} \quad (44)$$

If $\rho=1$, then

$$PF_e = \sqrt{\frac{1+\xi}{4}} = \frac{\sqrt{1+\xi}}{2} \quad (45)$$

In the figure 1, where the load only consume power according to a configuration in Y with neutral conductor, $P_\Delta=0$, then $\xi=0$. The power factor $PF_e = 1/2$. This result indicates that it is possible a subsequent series compensation that compensates the voltage in the load terminals in order to get a unit power factor, [4]. In fact, by means of an active compensator of series connection it is possible to obtain a balanced voltage set of positive sequence in the load terminals. In that way it would be gotten a $PF_e=1$.

The last expression of PF_e shows that before the series compensation, both power factors would be same if $\xi=3$. In that case both equivalent voltages have the same value. Regarding the load it does mean that the power P_Δ absorbed by this it would be $3P_Y$, or what is the same thing, the load would be constituted by the association of a star and a delta of resistances of the same value.

5. Conclusions

The theoretical of the electric power use two definitions of apparent power and their consequent definitions of power factor like magnitudes in confrontation. Nevertheless, both definitions can be conjugated from the point of view of the load compensation. Indeed, the apparent power of the European approach is related with a unit power factor for shunt compensation. On the other hand, the apparent power of the American approach is related with a unit power factor when it is possible a series/parallel compensation by means of a compensation equipment built by a combined power filters. This last power factor would be the appropriate figure of merit to measure the results of a universal compensation system. Nevertheless, the apparent power definition of the IEEE approach lack of a rigorous justification that avoids the unheard of some situations is.

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