Instantaneous Reactive Power Theory: A New Approach Applied to N Wire Systems

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Abstract. One of the most common strategies used in the Active Power Filters (APFs) control is the derived from the instantaneous reactive power theory. Since its publication, a lot of others formulations have been developed in order to achieve compensation objectives different of the proposed in the original one. Nevertheless, all those formulations present a common characteristic: none of them can be applied to n wire systems where n>4.

This paper presents a new approach which can be applied to n wire systems. The control strategy proposed in this new formulation can obtain any compensation objective. Besides, the control derived from the other formulations can be obtained by means of this new one.

Key words

Power Quality, Active Power Filter, Instantaneous Reactive Power Theory, Control Strategy.

1. Introduction

Along the last decades, several formulations of the instantaneous reactive power theory have been developed, [1]-[7]. Some of them treat the zero-sequence axe in a different way of the resting system. The other board all the components unified. Therefore, none of them can be applied to n wire systems with n>4.

In this paper a new formulation of the instantaneous reactive theory which can be applied to the n wire systems with n≥2 is presented. In this new formulation, denominated tensorial, the current vector is decomposed in two orthogonal components: the instantaneous active current which supply all the instantaneous real power and the instantaneous reactive power which supplies all the instantaneous imaginary power.

This is a global formulation which involves all the previously published to be applied to three-phase systems depending on the decomposition carried out to the current vector.

In the development of this new formulation, some mathematical operations are used, like the dyadic product or the exterior product. On the other hand, the instantaneous imaginary power gets a hemisymetrical tensor form, introduced at first time in the power electric systems study in 1986, [8]-[9].

The instantaneous imaginary power defined in the original formulation, [1], can only be applied to three-phase systems. Based on a coordinate translation, it can not be extended to n-phase systems. The definition presented in the modified p-q formulation or the proposed by the formulations based on synchronous reference frame can not be applied to n-phase systems.

This paper presents a definition of the instantaneous imaginary power applied to n- phase systems based on the suitable mathematical operations.

So, this paper is organized as follows. In section 2, the fundamental definitions are introduced, using the suitable mathematical operations. In section 3, the current vector is decomposed in two orthogonal components: the instantaneous active power and the instantaneous reactive power. In section 4, the expressions obtained in previous sections are particularized to a three-phase three-wire system and the strategy derived from the instantaneous reactive power theory is obtained. In section 5 simulation results are presented corresponding to a six-phase system whose load consists of a 12 pulses rectifier. In section 6 the most important conclusions are presented.

2. Basic Definitions

Based on Fig. 1, the instantaneous reactive power theory tensorial formulation defines the voltage vector as:

$$\vec{u} = \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix}^T \quad (1)$$

And the current vector as:

$$\vec{i} = \begin{bmatrix} i_1 & i_2 & \dots & i_n \end{bmatrix}^T \quad (2)$$

On the other hand, this formulation defines the instantaneous active power as:

$$p(t) = \vec{u}(t) \cdot \vec{i}(t) \quad (3)$$

and the instantaneous imaginary power as the order n hemisymetrical tensor **q(t)** which is calculated as the exterior product of current and voltage vector as follows:

$$\mathbf{q}(\mathbf{t}) = \vec{i}(t) \wedge \vec{u}(t) \qquad (4)$$

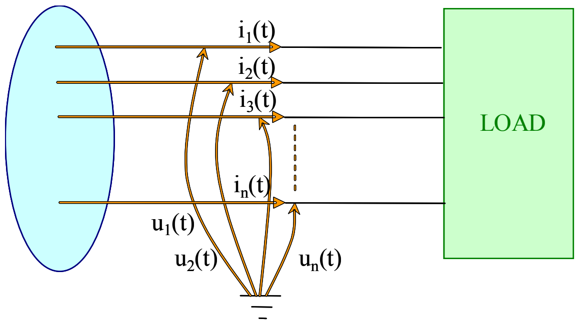

Fig. 1 n wire system

The exterior product is calculated as the difference of the next dyadic products:

$$\mathbf{q}(\mathbf{t}) = \vec{i}(t) \wedge \vec{u}(t) = \left(\vec{i} \otimes \vec{u}\right) - \left(\vec{u} \otimes \vec{i}\right) \qquad (5)$$

The dyadic product of current vector over voltage vector is defined as:

$$\vec{u} \otimes \vec{i} = \begin{bmatrix} i_1 & i_2 & \dots & i_n \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{bmatrix} = \begin{bmatrix} i_1 u_1 & i_1 u_2 & \dots & i_1 u_n \\ i_2 u_1 & i_2 u_2 & \dots & i_2 u_n \\ \dots & \dots & \dots & \dots \\ i_n u_1 & i_n u_2 & \dots & i_n u_n \end{bmatrix} \qquad (6)$$

And the dyadic product of voltage vector over current vector is defined as:

$$\vec{u} \otimes \vec{i} = \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \dots \\ i_n \end{bmatrix} = \begin{bmatrix} u_1 i_1 & u_1 i_2 & \dots & u_1 i_n \\ u_2 i_1 & u_2 i_2 & \dots & u_2 i_n \\ \dots & \dots & \dots & \dots \\ u_n i_1 & u_n i_2 & \dots & u_n i_n \end{bmatrix} \qquad (7)$$

So, the instantaneous imaginary power gets the next form:

$$\mathbf{q} = \vec{i} \wedge \vec{u} = \begin{bmatrix} 0 & i_1 u_2 - u_1 i_2 & \dots & i_1 u_n - u_1 i_n \\ u_1 i_2 - i_1 u_2 & 0 & \dots & i_2 u_n - u_2 i_n \\ \dots & \dots & \dots & \dots \\ u_1 i_n - i_1 u_n & u_2 i_n - i_2 u_n & \dots & 0 \end{bmatrix} \qquad (8)$$

3. Decomposition of the current vector

Firstly, the instantaneous active current, $\vec{i}_p$, is defined proportional to the voltage vector as follows:

$$\vec{i}_p = \frac{p(t)}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{\left(u_1 i_1 + u_2 i_2 + \dots + u_n i_n\right)}{\vec{u} \cdot \vec{u}} \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{bmatrix} \qquad (9)$$

expression equivalent to the next:

$$\vec{i}_p = \frac{\left(\vec{u} \otimes \vec{i}\right)}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{1}{\vec{u} \cdot \vec{u}} \begin{bmatrix} u_1 i_1 u_1 + u_1 i_2 u_2 + \dots + u_1 i_n u_n \\ u_2 i_1 u_1 + u_2 i_2 u_2 + \dots + u_2 i_n u_n \\ \dots \\ u_n i_1 u_1 + u_n i_2 u_2 + \dots + u_n i_n u_n \end{bmatrix} \qquad (10)$$

On the other hand, the current vector can be expressed as:

$$\vec{i} = \frac{\left(\vec{i} \otimes \vec{u}\right)}{\vec{u} \cdot \vec{u}} \vec{u} \qquad (11)$$

Therefore, the instantaneous reactive current is defined as follows:

$$\vec{i}_q = \frac{\mathbf{q}(t) \cdot \vec{u}}{\vec{u} \cdot \vec{u}} = \frac{\left(\vec{i} \wedge \vec{u}\right) \cdot \vec{u}}{\vec{u} \cdot \vec{u}} = \frac{\left[\left(\vec{i} \otimes \vec{u}\right) - \left(\vec{u} \otimes \vec{i}\right)\right] \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \qquad (12)$$

So, the load current vector is decomposed in two orthogonal components: the instantaneous active current proportional to the voltage and which carries the whole instantaneous real power and the instantaneous reactive current orthogonal to the voltage vector and which does not carry instantaneous active power, but instantaneous imaginary power.

The instantaneous real power is a real number and the instantaneous imaginary power is a n order hemisymetrical tensor where n is the phases number of the power system.

Several compensation objectives can be achieved applying this new formulation: instantaneous or average. Into the second kind of compensation, several targets can be considered: constant source current, unity power factor and balanced sinusoidal source current, [10]-[11].

4. Application to three-phase systems

Appling the tensorial formulation definitions to a three-phase three-wire system, Fig. 2, the current vector is expressed in the next way:

$$\vec{i}(t) = \vec{i}_p(t) + \vec{i}_q(t) =$$

$$= \frac{p(t)}{\vec{u} \cdot \vec{u}} \vec{u} + \frac{1}{\vec{u} \cdot \vec{u}} \begin{bmatrix} i_1 u_2 u_2 - u_1 i_2 u_2 + i_1 u_3 u_3 - u_1 i_3 u_3 \\ i_2 u_1 u_1 - u_2 i_1 u_1 + i_2 u_3 u_3 - u_2 i_3 u_3 \\ i_3 u_1 u_1 - u_3 i_1 u_1 + i_3 u_2 u_2 - u_3 i_2 u_2 \end{bmatrix} \qquad (13)$$

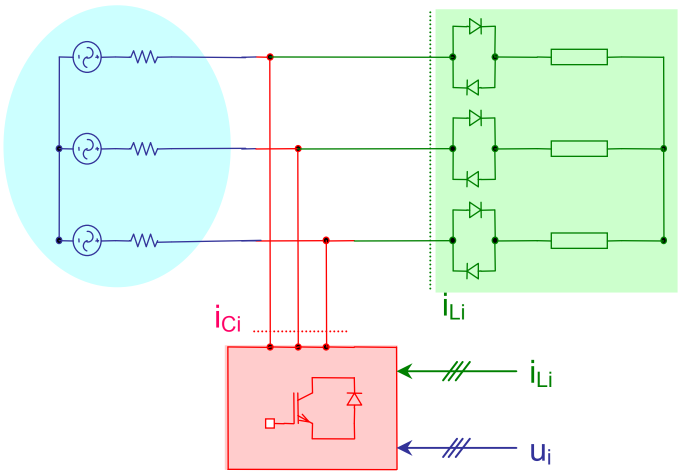

Fig. 2 Tree-phase three-wire power system

Therefore, the instantaneous imaginary power is expressed as:

$$\mathbf{q}=\begin{bmatrix} 0 & i_1u_2-u_1i_2 & i_1u_3-u_1i_3 \\ u_1i_2-i_1u_2 & 0 & i_2u_3-u_2i_3 \\ u_1i_3-i_1u_3 & u_2i_3-i_2u_3 & 0 \end{bmatrix} \tag{14}$$

This hemisymetrical tensor may be expressed as a vector in the next way:

$$\vec{q}=\begin{bmatrix} u_2i_3-i_2u_3 \\ i_1u_3-u_1i_3 \\ u_1i_2-i_1u_2 \end{bmatrix} \tag{15}$$

which is identical to the definition presented by the modified p-q formulation.

5. Original p-q formulation in the new framework

In the previous sections the modified p-q formulation is obtain from the tensorial approach. In the same way, the original p-q formulation equations are obtained in the present section. In fact, voltage and current are expressed in the 0αβ coordinates system. So, the voltage vector is:

$$\vec{u}=\begin{bmatrix} u_0 & u_\alpha & u_\beta \end{bmatrix}^T \tag{16}$$

where u_0 is the zero-sequence component, u_α the α component and u_β the β component.

On the other hand, the current vector is expressed as:

$$\vec{i}=\begin{bmatrix} i_0 & i_\alpha & i_\beta \end{bmatrix}^T \tag{17}$$

where i_0 is the zero-sequence component, i_α the α component and i_β the β component.

According to the p-q formulation, the zero-sequence component is submitted to an independent treatment. So, two other vectors can be defined: the αβ voltage vector or voltage vector without zero-sequence component, $\vec{u}_{\alpha\beta}$, and the αβ current vector or current vector without zero-sequence component, $\vec{i}_{\alpha\beta}$:

$$\vec{u}_{\alpha\beta}=\begin{bmatrix} 0 & u_\alpha & u_\beta \end{bmatrix}^T \tag{18}$$

$$\vec{i}_{\alpha\beta}=\begin{bmatrix} 0 & i_\alpha & i_\beta \end{bmatrix}^T \tag{19}$$

Therefore, the zero-sequence vectors can be defined. In fact, the voltage zero-sequence vector is expressed as:

$$\vec{u}_0=\begin{bmatrix} u_0 & 0 & 0 \end{bmatrix}^T \tag{20}$$

And the current zero-sequence vector as:

$$\vec{i}_0=\begin{bmatrix} i_0 & 0 & 0 \end{bmatrix}^T \tag{21}$$

As can be seen, the zero-sequence current vector and the αβ current vector are orthogonal and so on the respective voltage vectors.

Besides:

$$\vec{u}\cdot\vec{u}_{\alpha\beta}=\vec{u}_{\alpha\beta}\cdot\vec{u}_{\alpha\beta} \tag{22}$$

and:

$$\vec{u}\cdot\vec{u}_0=\vec{u}_0\cdot\vec{u}_0 \tag{23}$$

The αβ current vector can be decomposed in two orthogonal components by means of the tensorial develop. It is:

$$\begin{aligned}\vec{i}_{\alpha\beta}&=\frac{\vec{i}_{\alpha\beta}\otimes\vec{u}_{\alpha\beta}}{\vec{u}_{\alpha\beta}\cdot\vec{u}_{\alpha\beta}}\vec{u}_{\alpha\beta}= \\ &=\frac{\vec{u}_{\alpha\beta}\otimes\vec{i}_{\alpha\beta}}{\vec{u}_{\alpha\beta}\cdot\vec{u}_{\alpha\beta}}\vec{u}_{\alpha\beta}+\frac{\vec{i}_{\alpha\beta}\wedge\vec{u}_{\alpha\beta}}{\vec{u}_{\alpha\beta}\cdot\vec{u}_{\alpha\beta}}\vec{u}_{\alpha\beta}=\vec{i}_{\alpha\beta p}+\vec{i}_{\alpha\beta q}\end{aligned} \tag{24}$$

where $\vec{i}_{\alpha\beta p}$ defines the projection of the αβ current vector over the αβ voltage vector and is called αβ instantaneous active current and the $\vec{i}_{\alpha\beta q}$ represents the projection of the αβ current vector over an orthogonal voltage vector and is called instantaneous reactive current.

In fact, developing the αβ instantaneous active current presented in the equation (24), it is obtained the next:

$$\vec{i}_{\alpha\beta p}=\frac{p_{\alpha\beta}(t)}{u_{\alpha\beta}^2}\vec{u}_{\alpha\beta} \tag{25}$$

where $p_{\alpha\beta}(t)$ is the defined by Akagi et all, [1].

The current vector presented in equation (25) involves two components. The first one is the instantaneous active current in the axe α, $\vec{i}_{\alpha p}$, [1]:

$$\vec{i}_{\alpha p} = \frac{p_{\alpha\beta}(t)}{u_{\alpha\beta}^2} u_\alpha \qquad (26)$$

The second one is the instantaneous active current in the axe β, $\vec{i}_{\beta p}$, [1]:

$$\vec{i}_{\beta p} = \frac{p_{\alpha\beta}(t)}{u_{\alpha\beta}^2} u_\beta \qquad (27)$$

On the other hand, developing the $\vec{i}_{\alpha\beta q}$ expression in equation (24), it is obtained the next:

$$\vec{i}_{\alpha\beta q} = \frac{\vec{i}_{\alpha\beta} \wedge \vec{u}_{\alpha\beta}}{\vec{u}_{\alpha\beta} \cdot \vec{u}_{\alpha\beta}} \vec{u}_{\alpha\beta} = \frac{1}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} 0 \\ i_\alpha u_\beta^2 - i_\beta u_\alpha u_\beta \\ i_\beta u_\alpha^2 - i_\alpha u_\alpha u_\beta \end{bmatrix} = \frac{u_\alpha i_\beta - u_\beta i_\alpha}{u_{\alpha\beta}^2} \begin{bmatrix} 0 \\ -u_\beta \\ u_\alpha \end{bmatrix} \qquad (28)$$

The current expression shown in equation (28) involves two components. In fact, the α component is the corresponding to the original p-q formulation instantaneous reactive current in axe α, $\vec{i}_{\alpha q}$, [1]:

$$\vec{i}_{\alpha q} = -\frac{q_{\alpha\beta}(t)}{u_{\alpha\beta}^2} u_\beta \qquad (29)$$

The β component is the corresponding to the original p-q formulation instantaneous reactive current in axe β, $\vec{i}_{\beta q}$, [1]:

$$\vec{i}_{\beta q} = \frac{q_{\alpha\beta}(t)}{u_{\alpha\beta}^2} u_\alpha \qquad (30)$$

6. Simulation results

In this section, the definitions are applied to a six-phase system constituted by a balanced and sinusoidal source supplying a 12 pulses diode rectifier.

From this power system, the current two orthogonal components are calculated according to the definitions presented in section 4.

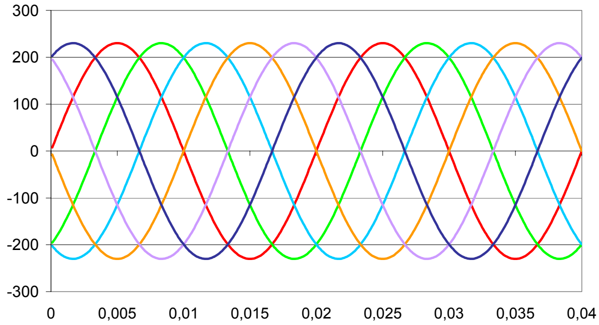

Fig. 3 Six-phase voltage waveform applied to the power system

Fig. 3 shows the voltage supplied by the source. A balanced a sinusoidal voltage system.

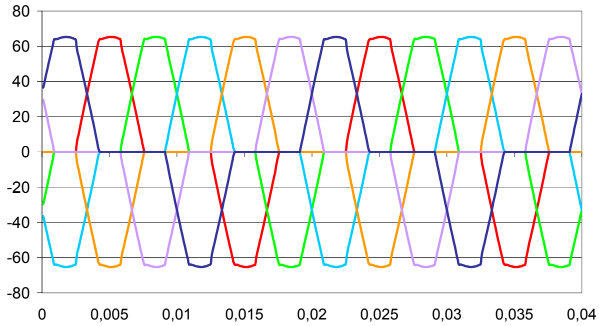

Fig. 4 Six-phase current consumed by the load

Fig. 4 shows the six-phase waveform corresponding to the current required by the load. Fig. 5 details the load current phase 1. It is a strongly non linear load.

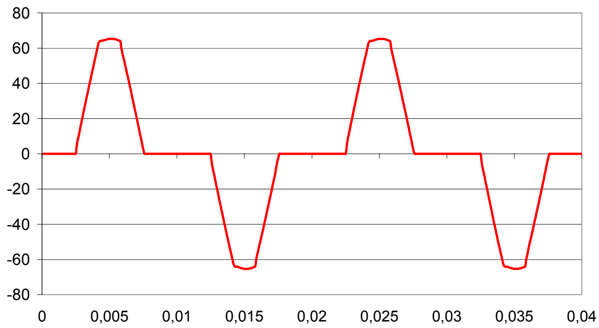

Fig. 5 Load current phase 1

Fig. 6 presents the phase 1 of the instantaneous active current. The corresponding six-phase waveform is balanced.

It carries the whole instantaneous real power as corroborated in Fig. 7 where the dot product $\vec{u}(t) \cdot \vec{i}_p(t)$ versus the product $\vec{u}(t) \cdot \vec{i}(t)$ is shown. These two graphs are superimposed. So, effectively, the instantaneous active current carries the whole instantaneous real power.

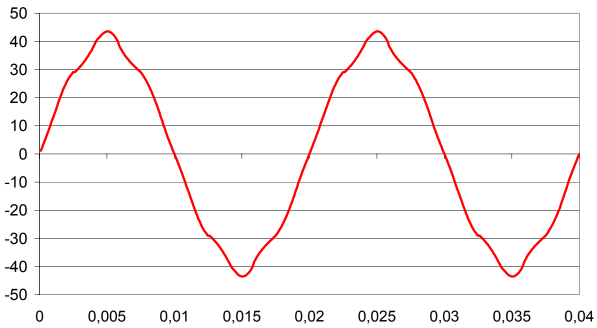

Fig. 6 Six-phase instantaneous active current

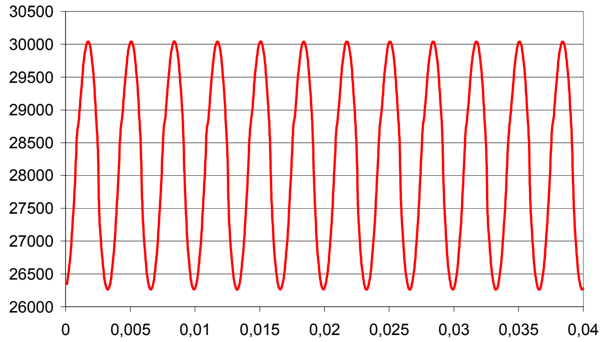

Fig. 7 Instantaneous active power

Fig. 8 presents the phase 1 of the instantaneous reactive current. The corresponding six-phase waveform is balanced.

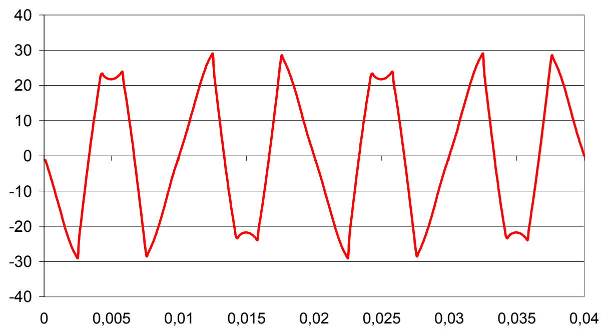

Fig. 8 Instantaneous reactive current phase 1

The instantaneous reactive current carries the whole instantaneous imaginary power. To corroborate it, Fig. 9 presents two graphs: one of them has been calculated as:

$$q = \sqrt{s^2(t) - p^2(t)} \tag{31}$$

where ‘s’ is the instantaneous apparent power calculated according to the next expression:

$$s(t) = \|\vec{u}(t)\| \|\vec{i}(t)\| \tag{32}$$

The another graph has been calculated as the norm of the n order hemisymetrical tensor defined in (8).

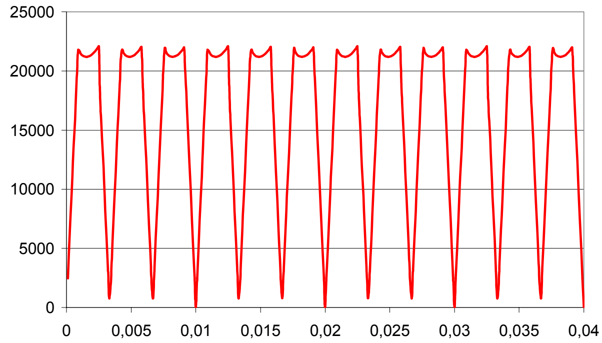

Fig. 9 Instantaneous imaginary power

Fig. 8 two graphs are superimposed, too. So, the instantaneous reactive current carries the whole instantaneous imaginary power.

7. Conclusion

In this paper, an instantaneous reactive power theory formulation has been presented. This new formulation proposes a way of calculating the instantaneous imaginary power which can be applied not only to three-phase power systems, but to n-phase power systems. Besides, the original formulation of the instantaneous reactive power theory is obtained in the framework defined by this new formulation. Finally, the paper presents simulation results, too, where all the calculations have been confirmed.

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