

Evaluation of excessive transmission line losses caused by unbalanced and nonlinear three-phase loads

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Abstract. This work evaluates excessive transmission line losses caused by unbalanced and nonlinear three-phase loads. The orthogonal decomposition of currents is applied in order to determine current component indispensable for energy transmission and current components that do not contribute to the energy transmission. The last ones cause excessive losses in the transmission lines which can be avoided.

Key words: transmission line losses, orthogonal current decomposition, asymmetrical and nonlinear three-phase loads

1. Introduction

This work evaluates excessive transmission line losses in the three-phase systems caused by unbalanced and nonlinear loads. In the single-phase systems, where currents and voltages contain exclusively fundamental harmonic components, energy transmission and reciprocal energy exchange between source and load can be adequately treated by the concept of active and reactive power [1]. However, this approach fails when currents and voltages contain higher order harmonic components or when unbalanced and nonlinear three-phase loads are analyzed. In such cases mathematical tools like orthogonal decompositions of currents in the time or frequency domains [2]- [8] can be applied. Using these tools, it is possible to determine current components indispensable for energy transmission and those current components that do not contribute to the energy transmission and cause excessive transmission losses which can be avoided.

In this work, the orthogonal decomposition of three-phase currents in the time domain is used to determine current component indispensable for energy transmission. The ratio between its RMS (root mean square) value and the RMS value of the original three-phase current is applied to introduce generalized power factor in the three-phase system. The generalized power factor is used to characterize the load unbalance and effects of nonlinear load. Excessive transmission losses in a low voltage distribution line are evaluated by case studies performed for different operating conditions.

2. Orthogonal decomposition of currents

In this section current and voltage vectors of the three-phase system are introduced together with the inner product. They are used to define the current vector component indispensable for three-phase energy transmission and the generalized power factor of the three-phase system.

Let $i_1(t)$, $i_2(t)$, $i_3(t)$ and $u_1(t)$, $u_2(t)$, $u_3(t)$ be the line currents and voltages of a three-phase system observed inside a selected time window $[0, T]$. Let us introduce the current vector $\mathbf{i}(t)$ and the voltage vector $\mathbf{u}(t)$ by (1).

$$\mathbf{i}(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix}, \quad \mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \quad (1)$$

The inner product of current and voltage vectors is defined by (2):

$$\begin{aligned} (\mathbf{u}, \mathbf{i}) &= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \mathbf{u}^T \mathbf{i} \, d\tau = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \begin{bmatrix} u_1(\tau) \\ u_2(\tau) \\ u_3(\tau) \end{bmatrix}^T \begin{bmatrix} i_1(\tau) \\ i_2(\tau) \\ i_3(\tau) \end{bmatrix} \, d\tau = \\ &= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p_1(\tau) + p_2(\tau) + p_3(\tau) \, d\tau = P_1 + P_2 + P_3 = P \end{aligned} \quad (2)$$

where p_1 , p_2 and p_3 denote instantaneous values of power in all three phases. From (2) it is obvious that the inner product of current and voltage vectors equals the average active power P of the three-phase system which is given as a sum of average active powers in individual phases denoted by P_1 , P_2 and P_3 .

Let us introduce RMS values of the current vector I and voltage vector U by Euclidean norms of these vectors (3) and (4). Squares of both norms equal the sums of squared current or voltage RMS values in individual phases.

$$I^2 = \|\mathbf{i}\|^2 = I_1^2 + I_2^2 + I_3^2 = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \mathbf{i}^T \mathbf{i} d\tau = \quad (3)$$

$$= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i_1(\tau)i_1(\tau) + i_2(\tau)i_2(\tau) + i_3(\tau)i_3(\tau) d\tau$$

$$U^2 = \|\mathbf{u}\|^2 = U_1^2 + U_2^2 + U_3^2 = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \mathbf{u}^T \mathbf{u} d\tau = \quad (4)$$

$$= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} u_1(\tau)u_1(\tau) + u_2(\tau)u_2(\tau) + u_3(\tau)u_3(\tau) d\tau$$

In (3) and (4) I_1 , I_2 , I_3 , and U_1 , U_2 , U_3 denote the RMS values of currents and voltages in all three phases. The equivalent conductivity of entire three-phase system G is defined by (5).

$$G = \frac{P}{U^2} = \frac{P}{\|\mathbf{u}\|^2} \quad (5)$$

The equivalent conductivity G (5) is required to introduce current vector \mathbf{i}_u (6) which is collinear with applied voltage vector \mathbf{u} and is the only current vector indispensable for energy transmission.

$$\mathbf{i}_u = G\mathbf{u} = \frac{P}{U^2} \mathbf{u} \quad (6)$$

The definition of current vector \mathbf{i}_u (6) is general. It is appropriate to be used in three-phase systems with unbalanced and nonlinear load. Considering (1), (2), (3) and (6), the norm (RMS) value of the current vector \mathbf{i}_u can be expressed by (7), which leads to (8).

$$\|\mathbf{i}_u\|^2 = (\mathbf{i}_u, \mathbf{i}_u) = \left(\frac{P}{\|\mathbf{u}\|^2} \mathbf{u}, \frac{P}{\|\mathbf{u}\|^2} \mathbf{u} \right) = \frac{P^2}{\|\mathbf{u}\|^2} \quad (7)$$

$$|P| = \|\mathbf{u}\| \|\mathbf{i}_u\| \quad (8)$$

The generalized power factor of the three-phase system PF' can be now defined by (9).

$$PF' = \frac{|P|}{\|\mathbf{u}\| \|\mathbf{i}\|} = \frac{\|\mathbf{u}\| \|\mathbf{i}_u\|}{\|\mathbf{u}\| \|\mathbf{i}\|} = \frac{\|\mathbf{i}_u\|}{\|\mathbf{i}\|} \quad (9)$$

In the first approximation the transmission line losses are proportional to the $\|\mathbf{i}\|^2$ while only current vector \mathbf{i}_u contributes to the energy transmission. The $\|\mathbf{i}_u\|^2$ is proportional to the transmission losses which cannot be avoided. Thus, the square of generalized power factor, actually $1/PF'^2$ (10), can be used as a measure for excessive transmission losses.

$$1/PF'^2 = \frac{\|\mathbf{i}\|^2}{\|\mathbf{i}_u\|^2} \quad (10)$$

3. Evaluation of excessive transmission losses

In order to evaluate excessive transmission losses in a low voltage distribution line a system with nonlinear and unbalanced load schematically shown in Figure 1 was applied. The feeder network is denoted by Q while C1 and C2 denote two measurement points.

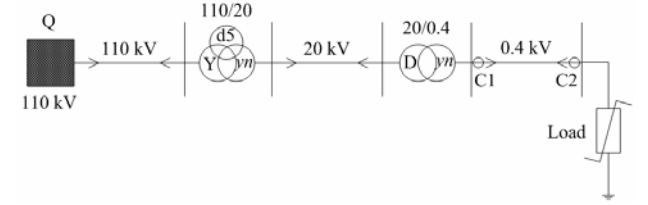


Figure 1: Schematic presentation of discussed system

Transmission losses were evaluated for the following cases:

1. Balanced linear load
2. Unbalanced linear load
3. Balanced nonlinear load
4. Unbalanced nonlinear load

The load power was 72.53 kW and 131.26 kW while the generalized power factor was between 0.73 and 1. The expression “unbalanced” load means that the RMS value of one line current is 20% higher than the RMS values of the other two line currents.

Figures 2 to 4 show the time dependent line voltages at the beginning u_{fb} (C1 in Figure 1) and at the end u_{fe} (C2 in Figure 1) of 0.4 kV line together with the corresponding line currents i_f .

Figure 2 shows the time behavior of line voltages and currents for a 72.53 kW linear load. The two upper graphs show currents and voltages for the balanced linear load, while the lower two graphs show the currents and voltages for the unbalanced linear load where the RMS value of one line current is 20% higher than the RMS values of the other two currents. The active load power is the same in both presented cases.

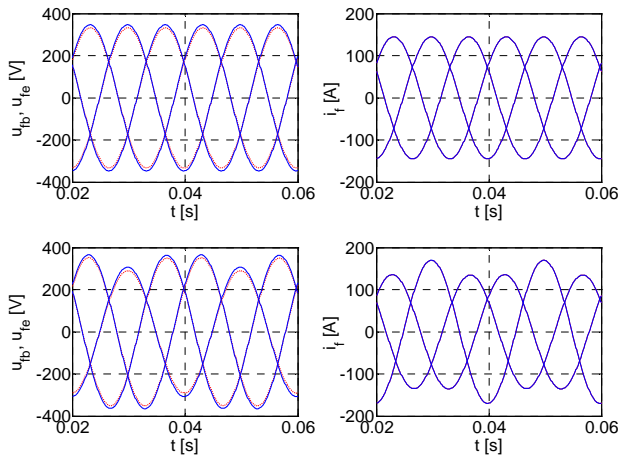


Figure 2: Time behavior of voltage and current given for balanced 72.53 kW linear load (upper graphs) and for unbalanced 72.53 kW linear load (lower graphs)

Figure 3 shows the time behavior of line voltages and currents for a nonlinear 72.53 kW load. The upper graphs are given for the balanced nonlinear load. The lower graphs are given for the unbalanced nonlinear load where the RMS value of one line current is 20 % higher than in the other two lines. The active load power is the same in both presented cases.

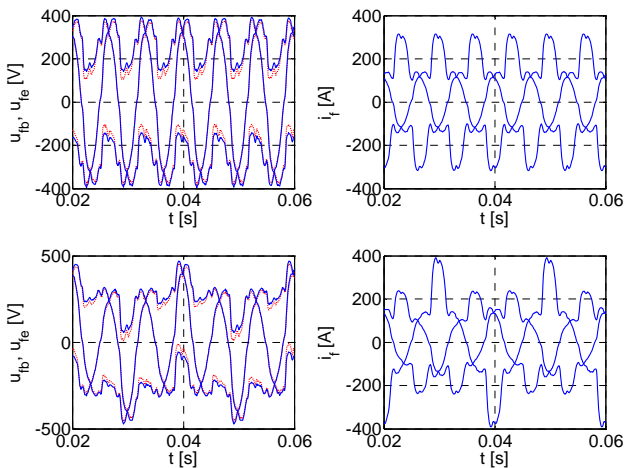


Figure 3: Time behavior of voltage and current given for balanced 72.53 kW nonlinear load (upper graphs) and for unbalanced 72.53 kW nonlinear load (lower graphs)

Figure 4 shows the time behavior of line voltages and currents for nonlinear unbalanced load. The upper graphs are given for load with active power 72.53 kW and generalized power factor 0.94, while the lower graphs are given for the load with active power 131.26 kW and generalized power factor 0.8.

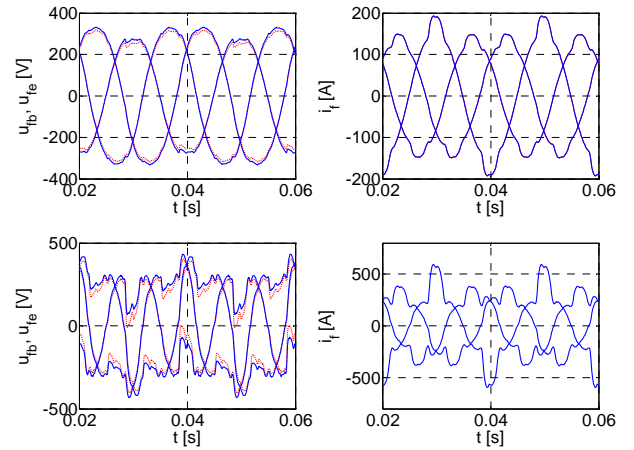


Figure 4: Time behavior of voltage and current given for nonlinear unbalanced load: upper graphs - active power 72.53 kW and generalized power factor 0.94; lower graphs - active power 131.26 kW and generalized power factor 0.8.

Table I shows the relative transmission losses P_{LOSS} calculated as a ratio $\frac{\|\mathbf{i}\|^2}{\|\mathbf{i}_d\|^2}$ (10) for balanced and unbalanced linear loads and balanced and unbalanced nonlinear loads. They are given for different values of the load active power P_{LOAD} and different values of generalized power factor PF' .

4. Conclusion

This paper evaluates excessive transmission line losses caused by unbalanced and nonlinear loads. In order to evaluate these losses in three-phase systems the RMS values of the total current vector and current vector component indispensable for power transmission must be determined first, which can be done by orthogonal decomposition of currents. The ratio between RMS values of the current vector and the current vector component indispensable for energy transmission can be used to evaluate excessive transmission line losses due to the unbalanced and nonlinear loads. It is shown that the excessive transmission losses increase with the inverse ratio of squared generalized power factor. According to the presented results, in the case of unbalanced nonlinear loads the excessive transmission losses can reach almost the same value as the transmission losses due to the linear and balanced loads, which means that the total transmission losses are almost doubled.

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Table I: Relative transmission losses P_{LOSS} given as a function of active load power P_{LOAD} and load generalized power factor PF' .

P_{LOAD} (kW)	Linear load				Nonlinear load			
	balanced		unbalanced		balanced		unbalanced	
	PF'	P_{LOSS} (pu)	PF'	P_{LOSS} (pu)	PF'	P_{LOSS} (pu)	PF'	P_{LOSS} (pu)
72.53	1.000	1.000	0.979	1.042	0.754	1.757	0.728	1.888
72.53	1.000	1.000	0.979	1.042	0.956	1.094	0.936	1.141
131.26	1.000	1.000	0.972	1.058	0.803	1.551	0.779	1.648