Abstract. Modelling wind power uncertainty is a critical aspect in the optimal management of power systems with high integration of this renewable resource. It is typically carried out by considering a limited number of representative scenarios that incorporate relevant properties such as hourly auto-correlation and diurnal forecasting profile. Considering a large amount of scenarios improves the wind power modelling, but increases the computational effort. To deal with this problem, a method to incorporate a big set of scenarios in stochastic unit commitment (UC) problem is presented in this paper. The effectiveness of the proposed methodology is evaluated by means of the analysis of a case study and the results are compared to those obtained from a stochastic programming method, concluding that the method presented in this paper offers an approximate solution in a reduced computational time.

Key words
Forecasting error; Mixed integer linear programming method; Scenario generation; Stochastic unit commitment.

1. Introduction

Effects of economic activities on the environment joined to high prices of fuels have alert governments of several countries around the world to increment the installed capacity of renewable power sources. However, one of the most important limitations in their integration is the uncertainty that these sources introduce in the managing of the power system. In other words, uncertainty of renewable sources represents a limitation to their power system management incorporation of energy storage and geographic characteristics [3], and inclusion of demand response programs [4], and incorporation of optimal scheduling methodologies considering uncertainty. The focus of this research work is developing a methodology for solving unit commitment (UC) problem incorporating large amount of scenarios to model wind power uncertainty.

Previously in [5] a stochastic model able to incorporate uncertainty of wind power generation, load demand, and system reliability based on stochastic mixed integer formulation was proposed. Scenarios of wind generation are generated by using autoregressive moving average (ARMA) model and scenario reduction process, while optimization problem incorporate “here-and-now” and “wait-and-see” decisions in the formulation.

In [6], weights in the objective function of optimization problem have been introduced; so that, they can be adjusted by system operator in order to obtain a robust solution in a reasonable computational time. The efficiency to solve optimization problem is improved by means of Benders’ decomposition algorithm. Scenario generation only takes into account a limited number of situations selected by using a reduction process; to compensate this problem, in [7] authors include a reserve margin for each scenario; it allows obtaining a robust solution. In [8], stochastic dynamic programming is applied to model the changes on wind power generation by means of a Markov process. In [9] and [10], economic dispatch problem is analytically solved incorporating the variability of wind generation and its impact on system load dispatch. Methodology presented is based on the analysis of probabilistic infeasibility incorporating wind power variability through Weibull probability distribution function (PDF) in the constraints. Then, Lagrange multiplier method is used to analyse the optimization problem taking into account several values of shape factor, scale factor, confidence level, and penetration factor. In [11], economic dispatch (ED) problem is analysed during a very short time interval and aggregated wind power production is considered. Based on this assumption, the concept of turbulence intensity is used to model short variations on wind power production.

In a similar way; in [12], Markov process are used to represent uncertainty on wind power generation from historical records; so that, variability is represented by means of discrete states instead of scenarios. Improving the results obtained from Monte Carlo Simulation (MCS) approach requires considering large amount of scenarios; so that, high computation resources are needed.

In this paper, a methodology for solving unit commitment incorporating large amount of scenarios is presented. In the proposed method of this paper, the probability of requiring a determined unit in a specific moment is determined. Then, a feasible solution to stochastic unit commitment is determined.

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The condition interval in some hour could be easily controlled using the forecasting error, it is used to complete the vector elements. This vector is used to evaluate if a determined scenario in a specific time is within the corresponding confidence interval. Factor parameter (2); this is summarized in (3):

\[ x_h = ax_h^{-1} + \beta \]  

(1)

where \( x_h \) is auto-correlated time series of scenario \( l (l = 1, 2, ..., L) \) at time \( h (h = 1, 2, ..., H) \), \( \alpha \) is the autocorrelation parameter, \( \beta \) is white noise with mean 0, standard deviation equal to \( \sqrt{1 - \alpha^2} \), and Gaussian PDF.

Hourly wind speed forecasting is normalized using (2):

\[ y_h = (W_h^b - \mu)/\sigma \]  

(2)

where \( y_h \) is the normalized wind speed forecasting, \( W_h^b \) is the forecasted wind speed at time \( h \), while \( \mu \) and \( \sigma \) are its mean and standard deviation, respectively. The shape of wind speed profile and auto-correlation are included in the scenario generation using the results obtained in (1) and (2); this is summarized in (3):

\[ z_l^h = x_l^h + y_h \]  

(3)

where \( z_l^h \) is the normalized wind speed of scenario \( l \) at time \( h \). Wind speed \( (WS_l^h) \) of scenario \( l \) at time \( h \) is generated by means of the probability transformation shown in Fig. 1 [13].

Those scenarios whose hourly values are out of the confidence level delimited by forecasting error are selected in order to be removed. Let (\( \delta \)) be significance level used to define maximum and minimum bound of forecasting error, it is used to complete the vector \( k_i^h \), which has \( H \) elements. This vector is used to evaluate if a determined scenario in a specific time is within the corresponding confidence interval. Namely, whether \( WS_l^h \) at time \( h \) is inside the confidence interval of this hour, the corresponding element of vector \( k_i^h \) becomes 1, in other case it becomes 0. A determined scenario \( l \) could not fulfill forecasting error condition in some hours; so that, parameter \( K_i \) is incorporated in order to evaluate the degree at which this scenario is within the corresponding confidence interval. Factor \( K_i \) is defined in (4):

\[ K_i = (\sum_{h=1}^{H} k_i^h) / H \]  

(4)

The condition \( K_i < 1 \) means that not all values of \( WS_l^h \) are between the corresponding confidence intervals, while the condition \( K_i \geq 1 \) means that scenario \( l \) is between confidence interval in all hour.

The amount of scenarios that are out of the confidence interval in some hour could be easily controlled using the factor \( \tau \); whether \( \tau \) is fixed to 0.7 those scenarios with values of \( K_i \) equal or higher than \( \tau \) are selected. Then those scenarios to be considered in stochastic UC problem are chosen by means of k-means clustering algorithm [14].

2. Methodology to Scenario Generation

In this paper scenarios of hourly wind speed are generated taking into account the auto-correlation, the profile of forecasted wind speed and the error of this prediction. In general, an initial set of scenarios are generated by using a first-order Markov process; then, some of these scenarios are selected by considering the forecasting error. The final step consists on choose the required amount of scenarios to be included in UC problem using k-means clustering algorithm. Autocorrelation is incorporated using (1):

\[ x_h = ax_h^{-1} + \beta \]  

(1)

where \( x_h \) is auto-correlated time series of scenario \( l (l = 1, 2, ..., L) \) at time \( h (h = 1, 2, ..., H) \), \( \alpha \) is the autocorrelation parameter, \( \beta \) is white noise with mean 0, standard deviation equal to \( \sqrt{1 - \alpha^2} \), and Gaussian PDF.

Hourly wind speed forecasting is normalized using (2):

\[ y_h = (W_h^b - \mu)/\sigma \]  

(2)

where \( y_h \) is the normalized wind speed forecasting, \( W_h^b \) is the forecasted wind speed at time \( h \), while \( \mu \) and \( \sigma \) are its mean and standard deviation, respectively. The shape of wind speed profile and auto-correlation are included in the scenario generation using the results obtained in (1) and (2); this is summarized in (3):

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Once, wind speed scenarios have been generated, the corresponding wind power production is calculated by using the modelling of the power curve of a single wind turbine presented in (5) [15]:

\[ WT_v = \begin{cases} 
(A + Bv + Cv^2)R_nN_t; & v_l \leq v \leq v_r \\
R_nN_t; & v_r \leq v \leq v_c \\
0; & v \leq v_l, v > v_c 
\end{cases} 
\]  

(5)

where \( v \) is wind speed, \( v_l \) is cut-in wind speed, \( v_r \) is rated wind speed, \( v_c \) is cut-off wind speed, \( R_n \) is rated power output, and \( N_t \) is number of wind turbines in the wind farm.

3. Unit Commitment and Proposed Method

In this section UC formulation is described besides of the proposed methodology to the incorporation of large amount of scenarios. UC problem consists on determining de generators to be committed to minimize the expected value of generating cost considering the variability of renewable resources and operating constraints related to the operation of the generating units such as maximum and minimum output power, operating ramp rate constraints; start-up and shut down ramp rate constraints, spinning reserve constraints and minimum up and down time constraints. In next sub-sections all this constraints are carefully explained.

A. Objective Function

Expected generating cost could be divided in fuel-consumption cost and starting up cost. Frequently, fuel consumption cost has been represented by using a quadratic expression that depends on the corresponding power generation. However, in this paper, fuel consumption cost has been simplified as a linear expression shown in (6):

\[ f = \sum_{l=1}^{L} P_l(l) \left( \sum_{n=1}^{N} a_n U_{n_l}^l + b_n P_{n_l}^l + C_{SUC}^l (1 - U_{n_l}^l)^n + U_{n_l}^l \right) \]  

(6)

where \( f \) is the expected value of total operating cost. \( P_l(l) \) is the probability of occurrence of a determine scenario \( l \). \( P_{n_l}^l \) is the power generation of unit \( n \) at time \( h \) in scenario \( l \). \( U_{n_l}^l \) is a binary variable to represent if generator \( n \), at time \( h \) is committed or not. \( C_{SUC}^l \) is the starting up cost of the generator \( n \) at time \( h \). Parameters \( a_n \) and \( b_n \) correspond to the fuel consumption of the generator \( n \). Starting up cost has been modelled as in (7):

\[ C_{SUC}^l = (HSU_n^{off} + OFF_n^{on} \leq MDT_n + CST_n) \]  

(7)

where \( HSU_n \) is hot startup cost, \( CSU_n \) is cold startup cost, and \( CST_n \) is cold start-up time of unit \( n \). \( OFF_n^{on} \) is an integer variable that counts the number of hours that generator \( n \) has been off. In a similar manner, \( ON_n^{on} \) counts the number of hours that generator \( n \) has been on.
The definition of these variables is presented in (8) and (9):

\[
ON^h_n = \begin{cases} 
ON^{h-1}_n + 1; & U^h_n = 1 \\
0; & U^h_n = 0 
\end{cases} \\
OFF^h_n = \begin{cases} 
OFF^{h-1}_n + 1; & U^h_n = 0 \\
0; & U^h_n = 1 
\end{cases}
\]

(8) (9)

B. Generation Limit Constraints

Power generation is limited between a minimum value \((P^\text{min}_n)\) and maximum value \((P^\text{max}_n)\), this constraint is presented in (10):

\[
P^\text{min}_n \leq P^h_n \leq P^\text{max}_h; \quad U^h_n = 1
\]

(10)

C. Operating Ramp Rate Constraints

Ramp constraint models the limitations of the conventional generators to change their power generation, these limitations are modelled as is shown in (11):

\[
p^h_{m,l} - p^h_{m,l-1} \leq UR_m; \quad U^h_n = 1; \quad U^h_n = 0 \\
-p^h_{m,l} - p^h_{m,l-1} \leq DR_m; \quad U^h_n = 1; \quad U^h_n = 1
\]

where \(UR_m\) and \(DR_m\) are the ramp up and ramp down rates of generator \(n\), respectively.

D. Startup and Shutdown Ramp Rate Constraints

Ramp limitations during the starting of a determined unit \(n\) are considered in (12):

\[
p^h_{m,l} \leq SUR_m + p^\text{min}_n, \quad U^h_n = 1; \quad U^h_n = 0 \\
-p^h_{m,l} \leq SDR_m + p^\text{min}_n, \quad U^h_n = 1; \quad U^h_n = 1
\]

where \(SUR_m\) and \(SDR_m\) are the startup and shut down ramp rates, respectively.

E. Spinning Reserve Constraints

Reserve margin is a specification that allows system operator face unexpected failures in any unit of the generation system. This margin is adjusted by means of parameter \(SR\) in (13):

\[
\sum_{n=1}^{N} \sum_{m=1}^{M} p^h_{m,l} | U^h_n | - \sum_{n=1}^{N} P^\text{min}_n | U^h_n | \geq (SR) D^h; \quad U^h_n = 1, U^h_n = 1
\]

(13)

F. Minimum Up and Down Time Constraints

Conventional generators have to be on or off for at least a determined number of hours that depends on the technology of each unit. These limitations are modeled including minimum up time \((\text{MUT}_n)\) and minimum down time \((\text{MDT}_n)\) of generator \(n\). This constraint is presented in (14):

\[
ON^h_n \geq \text{MUT}_n \\
OFF^h_n \leq \text{MDT}_n
\]

(14)

The methodology proposed in this research work consists on solving each scenario separately in order to reduce the computational efforts to obtain a feasible solution. This task is carried out by analyzing each scenario separately using MILP formulation proposed in reference [16], which allows determining the probability of requiring a determined generator in a specific time step; this is expressed in (15):

\[
P_r(U^h_n = 1) = \sum_{l=1}^{L} P_r(l) U^h_n
\]

(15)

Once the \(P_r(U^h_n = 1)\) has been estimated, those hours that have high probability of be committed are chosen.

The criterion used to select these hours is based on their corresponding probabilities, considering a determined significance level \(\theta\); if the condition \(P_r(U^h_n = 1) \geq \theta\) is fulfilled, then generator \(n\) is committed at time \(h\).

The resulting solution could not be feasible; so that, this is repaired using minimum up/down time repairing frequently used in Priority List UC method in order to be feasible. Repairing mechanism used here is that proposed in reference [17].

Solving ED problem, expected cost and probability of loss of reserve margin \((LRM)\) are evaluated to determine the characteristics of the obtained solution. The general procedure is presented in Fig. 2.

4. Case Study

The proposed method to the solution of UC problem incorporating the uncertainty related to the wind power generation is illustrated by analysing the power system whose characteristics are presented in Table I and Table II [16], while Table III presents hourly wind speed forecasted and the corresponding error. Forecasting error was estimated from results presented in reference [18] using ARMA model. The corresponding wind power scenarios were determined by modelling a single wind turbine with \(P_r = 2.5\) MW, \(v_i = 12\) m/s, \(v_r = 12\) m/s, \(v_e = 25\) m/s, and \(N_t = 80\).

Load demand is that considered in [16], while spinning reserve considered was 0.1. Probability of occurrence of a determined scenario is considered equal for all of them. The number of scenarios considered was \(L=200\). The proposed method was implemented in MATLAB and GAMS programming languages using CPLEX solver, the computer used is provided of Intel (R) Core (TM) i7-3630QM CPU @ 2.40 GHz with 8.00 GB of memory and 64 bit operating system.

Fig. 3 shows wind power scenarios considered, which has high level of uncertainty due to the forecasting tool considered. Table IV presents the PDF of commit a determined unit in a specific moment; the role of each generating unit could be easily recognized from this PDF; so that, those generators with probability equal to 1 correspond to base or cycling role; while those with lower probability correspond to peak role.
### Table I. - Description of the 10-Units Power System (Part 1)

<table>
<thead>
<tr>
<th>$n$</th>
<th>$P_{\text{min}}$ (MW)</th>
<th>$P_{\text{max}}$ (MW)</th>
<th>$a_n$ ($/h)$</th>
<th>$b_n$ ($$/MWh)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>455</td>
<td>959.82</td>
<td>16.480</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>455</td>
<td>944.05</td>
<td>17.448</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>162</td>
<td>690.80</td>
<td>16.900</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>130</td>
<td>670.30</td>
<td>16.817</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>130</td>
<td>421.52</td>
<td>20.444</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>80</td>
<td>354.410</td>
<td>22.972</td>
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<tr>
<td>7</td>
<td>20</td>
<td>80</td>
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<td>27.827</td>
</tr>
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<td>8</td>
<td>25</td>
<td>85</td>
<td>656.370</td>
<td>26.188</td>
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<tr>
<td>9</td>
<td>25</td>
<td>85</td>
<td>663.050</td>
<td>27.414</td>
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<td>10</td>
<td>10</td>
<td>55</td>
<td>668.480</td>
<td>27.902</td>
</tr>
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### Table II. - Description of the 10-Units Power System (Part 2)

<table>
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<tr>
<th>$n$</th>
<th>$I_{S_n}$ (h)</th>
<th>$MUT_{S_n}$ (h)</th>
<th>$MDF_{S_n}$ (h)</th>
<th>$CSC_{S_n}$ ($)$</th>
<th>$HSC_{S_n}$ ($)$</th>
<th>$CST_{S_n}$ (h)</th>
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<td>8</td>
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<td>4500</td>
<td>5</td>
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<tr>
<td>2</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>10000</td>
<td>5000</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>-6</td>
<td>6</td>
<td>6</td>
<td>1800</td>
<td>900</td>
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<td>560</td>
<td>4</td>
</tr>
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<td>5</td>
<td>1100</td>
<td>550</td>
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<td>30</td>
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</table>

### Table III. – Wind speed forecasting

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Wind (m/s)</th>
<th>Error (%)</th>
<th>Time (h)</th>
<th>Wind (m/s)</th>
<th>Error (%)</th>
</tr>
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<td>39.38710</td>
<td>13</td>
<td>10.5</td>
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<td>57.13166</td>
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<tr>
<td>3</td>
<td>3.1</td>
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<td>15</td>
<td>10.5</td>
<td>32.09627</td>
</tr>
<tr>
<td>4</td>
<td>2.2</td>
<td>105.50740</td>
<td>16</td>
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<td>32.58406</td>
</tr>
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<td>33.04227</td>
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<td>120.13370</td>
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<td>12</td>
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<td>30.40973</td>
<td>24</td>
<td>10.5</td>
<td>35.64860</td>
</tr>
</tbody>
</table>

### Fig. 3. Wind power scenarios

The error of the solution obtained from the proposed method is 0.681%; however, it was obtained in a reduced computational time.

### Fig. 4. Generation cost vs. significance level $\theta$

Fig. 4 shows how changes the expected cost and the probability of loss reserve margin ($Pr\{LRM > 0\}$) for several values of significance level $\theta$. It is possible observing how as $\theta$ increases, a cheaper solution is obtained; however, the probability of loss reserve increases due that less generating capacity is committed.

Table IV presents the solution considering $Pr\{LRM > 0\} = 0.000208$ and $\theta = 0.05$. The comparison to UC solution obtained from stochastic programming method [19] was carried out; the solution obtained from stochastic programming method was $531,111.8\$ in 3,095.193 seconds, while the cost that corresponds to solution presented in Table V is $534,732.3\$, obtained in 1,918.706 seconds.

### 5. Conclusion

This paper presented a methodology to solve the UC problem to be applied in systems with high integration of renewable power sources. The proposed methodology consisted on the generation of some representative scenarios which were selected considering the auto-correlated nature, the hourly wind speed forecasting and its corresponding error. In the next step, the PDF of a determined generator be committed or not is estimated by solving each scenario using MILP formulation. Finally, according to a determined probability level, those hours with probability of occurrence equal or higher than $\theta$ are selected and the minimum up/down time repairing is applied in order to obtain a feasible solution. The proposed methodology was illustrated through a study case whose results were similar to those obtained by the application of a stochastic programming method in a reduced computational time.

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References