Compensation of Discontinuous Conduction in
Three Phase Voltage Source PWM Inverters

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Abstract. Nowadays the increasing utilization of alternative energy sources demand power converters capable of delivering high quality energy to the low voltage grid. Good dead time compensation and linear operation are essential to keep low order harmonics at acceptable levels. High-current operating conditions are well handled by traditional methods, but low current behaviour and waveforms with multiple zero crossings within a switching period are problematic, especially at low load conditions at or below 10\% load. Such conditions often occur in the battery chargers of electric cars. Power quality is also essential for sensorless drives, as the accuracy of the output voltage is very important especially at low rotational speeds. This paper describes the identified cases of discontinuous conduction during dead time. Based on the voltage and current waveforms of the three phase inverter, an improved dead time compensation method is shown for symmetric PWM controlled inverters. The compensator uses already available measurement data, and compensates the resulting error voltage at duty cycle level. The described method can be used for any type of three phase modulation. A similar approach can also be used for two phase modulations.

Key words
PWM, inverter, linearization, discontinuous, dead-time

1. Introduction

In Symmetric PWM controlled voltage sourced inverters, the switch controller always drives both controllable switches in a phase leg to the off state for a small time interval during switch-over to avoid shoot-through. This blanking time is always more than what is needed for the switches to turn off for safety reasons. The remaining time is called effective dead time, and is marked with $t_d$ in this paper. During $t_d$ only the anti-parallel diodes can conduct the output current. The presence of effective dead time can introduce output waveform distortion and harmonic content in the output current of inverters. The effects become more severe at increased switching frequencies.

This paper summarizes the problem of dead time at low currents, and compares the existing compensation methods. It describes the effect of discontinuous conduction during dead time. A compensation method for this has already been described for single phase inverters in [1].

This article tries to extend this method for general three phase inverters, as seen on Fig. 1. This widely used circuit does not have a neutral connection. This, together with the fact that switch-over may happen simultaneously in more than one phase leg, makes the three phase compensation algorithm more complex than the single phase one.

2. Three phase signals

In a three phase system the phases can be put in order at any moment based on the ideal duty factor values as in (1). It will be shown, that it is useful to keep this order during the whole calculation. Usually $u_1 \leq u_2 \leq u_3$ but not necessarily as the output filter also has voltage drop.

$$D_1 \leq D_2 \leq D_3$$ (1)

Voltage and current waveforms of the circuit on Fig. 1. with a symmetric three phase PWM modulation can be seen on Fig. 2. at an arbitrarily chosen phase angle.

On Fig. 1. and Fig 2. the voltage between the neutral point (N on Fig 1.) and the ground point of the DC bus is the neutral voltage marked with $u_N$. 
Based on simple circuit theory and the fact that the equations in (2) are always true for the phase voltages of a symmetrical star, we get (3) for continuous conduction. $u_{bA}$, $u_{bB}$, $u_{bC}$ are the output voltages of the phase legs. The result of this is a four step signal [9] drawn on Fig 2. If one phase leg goes discontinuous, the formula will be different. This can be seen for example for phase “a” in (4).

\[
\begin{align*}
u_{bA} + \nu_{bB} + \nu_{bC} &= 0 \\
i_{bA} + i_{bB} + i_{bC} &= 0 \\
u_{NC} &= \frac{u_{bA} + u_{bB} + u_{bC}}{3} \\
u_{ND} &= \frac{u_{bA} + u_{bB} + u_{bC}}{2}
\end{align*}
\] (2)

The switching frequency component of each inductor voltage is determined by the potential difference between the bridge voltage of the phase ($u_{bP}$) and the neutral point ($u_N$). These have been plotted on Fig 2. The phase voltages do not change quickly and can be treated as constant for one switching period. The average voltage of the inductors will be close to zero for one switching period. Thus the thick black lines on the $u_{bP}$ - $u_N$ diagrams in Fig 2. can be treated as zero axes of the inductor voltage diagrams. From these, the current waveforms seen in Fig 2. can already be drawn.

On the current curves, $\Delta I_{1,2,3}$ marks the difference between the inductor current values measured at the two switch-over points. The maximum and the minimum current points are not necessarily at switch-over. Because of this, $\Delta I_P$ is not necessarily equal to the peak to peak current ripple of phase P. Still, because of the symmetry of the waveforms on Fig 2., the average current is equal to the current at switch over plus / minus the half of the current ripple $\Delta I_P$. This means that the same equations from [1] can also be used for general three phase inverters, if the new $\Delta I_P$ values seen on Fig 2. can be determined accurately. This can be done from available data similar to what is seen in (5). The exact method also calculates with the changes of the base harmonic current within a switching period, and it has been tested with primitive dead time compensation algorithms in [10].

\[
\begin{align*}
\Delta I_1 &= -\frac{u_1}{L} \cdot t_{1H} \\
\Delta I_2 &= \frac{(-u_2) \cdot t_{1H} + \left(\frac{2}{3} u_{dc} - u_2\right) \cdot (t_{2H} - t_{1H})}{L} \\
\Delta I_3 &= \frac{u_3}{L} \cdot t_{3L}
\end{align*}
\] (5)

Fig 2. Voltages and currents of an ideal three phase inverter.

Fig 3. Drive signals and output voltage of a phase leg.
3. Continuous Conduction and Dead Time

The control signals and the voltage of a phase leg can be seen on Fig. 3. During effective dead time, the output voltage of the bridge \(u_b\) is determined by the direction of the output current because of the free-wheeling diodes. This will cause an error voltage on the output which needs to be compensated for [2]. Most software based compensation methods, such as the ones described in [2] [3] and [4] only compensate for these two cases.

From Fig. 3, it can also be seen that if the current waveform is continuous and has a zero crossing, than the volt-seconds lost during one switch-over (which happens during positive current) are gained back during the next switch-over (during negative output current) within the same switching period. This state of operation is taken into account in some compensation methods, like in [4].

There are methods which try to solve the problem by using extra detection hardware for each switching element as described in [5] and [6]. These can detect the momentary current direction when a switch-over is initiated. This detection is also problematic for current waveforms having multiple zero crossings within one switching period.

4. Discontinuous Conduction

Discontinuous conduction happens in a phase leg if its output current reaches zero during effective dead time. If this happens, the free-wheeling diode will cease conduction, and the phase voltage plus the voltage of the neutral point \(u_n\) will be seen on the phase leg. Four such discontinuous cases are possible, as described in [1]. Figures 4. and 5. show the two cases for switching from \(S_L\) to \(S_H\) (a1 and a2) and two for switching from \(S_H\) to \(S_L\) (b1 and b2). \(t_z\) is the time required for the current to reach zero after switch-off. Discontinuous conduction happens only if (6) is satisfied. The current remains zero through the rest of the dead time, until the opposing switch is turned on.

\[
0 < t_z < t_d \quad (6)
\]

A. Preliminary Conditions

All calculations are completely analogous for the "a" and the "b" cases. Because of this, the calculations will only be shown for the a1 and a2 cases.

The equations for determining the effect of discontinuous conduction during dead time were tested in [1] for single phase inverters. These can also be used for three phase converters because of the symmetry of the current waveforms described in section 2.

The output of the current controller in an inverter is the duty factor \(D\) for each phase. In the ideal case, this means that \(t_{HI} = D*T\) and \(t_{LO} = (1-D)*T\). The goal of any dead time compensator is to make sure that the average phase leg voltage meets this expectation, as seen in (7).

\[
D \cdot u_{dc} + (1-D) \cdot (-u_{dc}) \equiv \frac{1}{T} \int_0^T u_{in} \, dt \quad (7)
\]
\[ I_{\text{ph,av}} = I_{L,av} = \frac{1}{T} \int_0^T i_t(t) \, dt \] (8)

The output voltage after the L filter inductor of a phase leg is equal to the phase voltage plus the voltage of the neutral point \((u_{N})\). The output current of the inverter is equal to the average current of the inductor \((8)\).

B. Time To Zero and Error Voltage in the a1 Case

By using the definition of \(\Delta I_p\) from Fig. 2 in section 2, and omitting the discontinuous time interval from \((8)\), the result is \((9)\). \(I_{\text{off}}\) is the current at switch-off, which happens when a switch-over is initiated. The time to zero for the a1 case can be expressed as in \((10)\).

\[ I_{P,av} = \frac{1}{T} \left( I_{\text{off}} + \frac{\Delta I_p}{2} \right) \left( T - t_d + t_d \right) \] (9)

\[ t_c = \frac{-I_{\text{off}} \cdot L}{u_{L,av}} \quad \text{where} \quad u_{L,av} = \frac{u_{DC}}{2} - u_{NS} - u_p \] (10)

\(u_{L,av}\) in \((10)\) is the voltage across the inductor in the actual P phase leg. \(u_p\) is the phase voltage, \(u_{NS}\) can be calculated from \((3)\) by substituting \(u_{off}/2\) for the voltage of the phase leg under investigation, because the output current is negative during \(t_c\), as seen on Fig. 4.a1. From \((9)\) and \((10)\), \(I_p\) can already be expressed. This quadratic equation can be solved as in \((11)\). Then \(t_1\) can be determined \((10)\).

\[ a = \frac{L}{u_{L,av}} \]
\[ b = a \cdot \frac{\Delta I_p}{2} - T + t_d \]
\[ c = \frac{\Delta I_p}{2} (T - t_0) + I_{L,av} \cdot T \] (11)

The above calculation was based on the assumption, that \(u_0\) does not change while the current decreases to zero. This does not necessarily apply in a symmetrical voltage system, as two switch-overs can overlap. In the a1 case this can happen if the duty factor of the actual phase leg under investigation is not the smallest in the order in \((1)\). In this case, the \(t_1\) value needs to be calculated from \((12)\). If \(t_1 < t_d\) then there is a switch-off in the phase leg with smaller D. A diode will take over conduction in that phase leg after \(t_1\) time. Thus the new \(u_{NS}\) has to be determined from \((3)\) based on the direction of the current in that phase. The resulting \(u_{NS}\) is used in \((13)\).

\[ t_1 = \frac{D_{\text{off}} - D_s}{2t_s} \] (12)
\[ u_{L,av} = \frac{u_{DC}}{2} - u_{NS} - u_p \] (13)

As \(u_N\) changes, the voltage across the inductor will also change \((13)\), and the a1s case will happen which can be seen on Fig. 6.

After \(t_1\) time (marked with \(\oplus\) on Fig. 6.) the slope of the current changes. The new time-to-zero value can be calculated the same way, based on \((9)\), but \((13)\) and \((14)\) are needed instead of \((10)\). The result of this new system of equations can be seen on \((15)\).

\[ t_{\text{cs}} = \frac{I_{\text{off}} \cdot L}{u_{L,av}} \] (14)

\[ a = \frac{L}{u_{L,av}} \]
\[ b = a \cdot \frac{\Delta I_p}{2} - T + t_d - \frac{1}{u_{L,av}} \cdot u_{L,av} \]
\[ c = \frac{\Delta I_p}{2} \left( T - t_0 + \frac{1}{u_{L,av}} \cdot u_{L,av} \right) + I_{L,av} \cdot T \] (15)

So if \(t_1 < t_d\) then \(t_{\text{cs}}\) shall be overwritten with \(t_{\text{cs}}\) calculated from \((14)\). After finishing the above calculations, the equation in \((6)\) can be checked to see if the a1 case is applicable. If yes, then the corresponding error voltage can be determined for this case. This is done very similarly to \([1]\), the main difference is in the way of determining the output voltage of the inverter during the discontinuous period. \((4)\) is required for this, but the three phase formula also has to be able to handle a switch-over in the next phase leg with smaller duty factor. This happens if \(t_2 < t_1 < t_3\). In this case, a diode will take over conduction in that phase leg after \(t_1\) time. Thus the new \(u_{NS}\) has to be determined from \((4)\) based on the direction of the current in that phase. The error voltage formula in \((16)\) handles this case, but it can also be used for the basic case if \(t_{\text{cs}} = t_{\text{cs}}\) is substituted.

\[ \Delta u = \frac{t_0 - t_1}{T} \left( u_{N,3} + u_p - \frac{u_{DC}}{2} \right) + \frac{t_1 - t_0}{T} \left( u_{N,2} + u_p - \frac{u_{DC}}{2} \right) \] (16)

C. Time To Zero and Error Voltage in the a2 Case

Based on \((8)\), as seen in \([1]\), the time-to-zero can be expressed as in \((17)\).
The calculated error voltage for the a2 case is also valid only if (6) is satisfied. It can also be seen for the a2 case that if $I_{L_{av}} > 0$ and $t_z > t_d$ then the conduction is continuous and there are no zero crossings in the current waveform.

The resulting error voltage for the a2 case is expressed in (18). This formula is also capable of handling a switch-over in the phase with smaller D during dead time, similar to (16). This happens if $t_z < t_1 < t_d$. If the D of the actual phase under investigation is the smallest, or $t_1 > t_d$ then a $t_z = t_1$ substitution can be used. For $t_1 < t_z$ then a $t_1 = t_z$ substitution gives the correct results.

\[
\Delta u = -u_{dc} \frac{t_1}{T} + \frac{t_1 - t_1}{T} \left( u_{N1} + u_p - \frac{u_{dc}}{2} \right) + \frac{t_2 - t_1}{T} \left( u_{N2} + u_p - \frac{u_{dc}}{2} \right)
\]

\[
(18)
\]

**D. Dead Time Compensation**

A dead time compensator based on the approach described above can get the value of the ideal duty cycle D from the current controller. Its task is to calculate the real D' duty cycle which will be applied to the PWM module using inputs from sensors in the converter. This can be done based on (19).

\[
D \cdot u_{dc} = D' \cdot u_{dc} + \Delta u
\]

\[
(19)
\]

**5. Simulation results**

In the previous sections, the average error voltages caused by dead time and discontinuous conduction have been determined for the “a” cases. As the “b” cases are analogous to this, the same steps were used to determine the corresponding formulae. The compensator has been tested in Matlab Simulink on a general three phase inverter model. The modelled main circuit is almost identical to Fig. 1. The load of each phase leg is a series RL element and an ideal three phase voltage source connected in star. The model of the semiconductor switches is capable of modelling dead time behaviour and discontinuous conduction. The voltage drop of the semiconductors has not been modelled. A zero crossing detector has been added to the model which forces zero voltage across the inductor if the current has reached zero during dead time. This was required to avoid oscillations because of the discrete time steps of the simulation. The sorting of the phases based on duty factor (1) and the calculation of the $\Delta I$ values (5) has been done as detailed in [10]. The actual compensator containing (6) and everything from (9)-(18) has been realized in a Matlab script instead of Simulink modelling, to have a more clear-cut view on the complex set of equations and conditions.

A linear interpolation based dead time compensator seen on Fig. 7. has been used for reference using $I_{thr} = \Delta I/2$.

The current controller used in the simulation uses Park transformation and two separate PI controllers for D and Q currents. A symmetrical / space vector modulator was also used. The simulation was run with settings for 400V 50Hz three phase grid connected operation. The component values were calculated for 50A nominal peak output phase current, using an inductor of 5%, and a series resistor of 1% nominal load impedance. In the simulation, the carrier frequency was fixed at 16kHz and the effective dead time was 3µs.

Testing is especially interesting at low current levels, where the $\Delta I$ values are comparable to the amplitude of the base harmonic current. The model has been tested at 10% load. Results using the linear approximation based dead time compensator can be seen on Fig. 8. A THD of 9.3% was reached with this method. The results with the new discontinuous method at the same operational conditions are on Fig. 9. This resulted in a THD of 2%.

**6. Conclusion**

In this paper it was shown that the discontinuous method of dead time compensation is applicable for three phase systems. A complex simulation model has been built. The resulting difference in THD compared to simpler methods is quite significant, especially at low load conditions or in low frequency drives.

There are two problems with the new method. First, the complex algorithm requires numerous arithmetical operations, most of which has to be performed within a switching period. Simplifications are needed to decrease the number of divisions. As the cost of computational power keeps decreasing, this will be a minor problem. The other problem is that the accurate value of the effective dead time is required as an input, like in [3] and in [5]. This value depends on conditions like temperature, DC bus voltage or output current. There are several methods to obtain this. For example, it can be obtained from a look up table which can be filled during a calibration procedure. A similar approach is done from output current using empirical formulas in [8]. A less costly way is to use an integrator in the feedback loop, which continuously "learns" the required values. A similar approach can be seen in [4].
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References


Fig. 8. Simulation results with the linear approximation method. Top to bottom: voltage and current waveforms, error voltage used for compensation, and output current spectrum. THD = 9.3 %

Fig. 9. Simulation results with the discontinuous method. Top to bottom: voltage and current waveforms, error voltage used for compensation, and output current spectrum. THD = 2 %