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Abstract.

With the energetic modelling, we search to make a tool with which to predict a future spending of energy in different races and optimize this energy.

The energetic modelling of Aníbal vehicle is very important for our objectives because it provides a tool that is very useful for our designs. Thanks to modelling we can save time and money because we can make the test of our ideas and designs before spend our resource. Also the modelling is very useful for our pilots due to before the race they can test different ways of driving for to save the maximum energy possible.

Results are very good, in the different graphics we can see that the current, power and energy in the time is very similar to real graphics.

Key words

Energetic modelling, Solar race car, photovoltaic, renewable energy, energy efficiency, electric car.

1. Introduction

A solar race car is an electric vehicle that is lightweight and aerodynamically efficient and it uses the irradiance from sunlight using photovoltaic system to generate its own power for propulsion and for storing in batteries. So with renewable energy this prototype car is self-sufficient and does not damage the environment with pollutions [1].

The electric propulsion system idea in a car is not new. The first electric car was developed in 1838, and there were more electric cars in the streets than gasoline cars until 1930’s [1].

There are different challenge for solar vehicles and others experimental cars. And the “SOLAR RACE Región de Murcia” is one of the most famous in Spain. And it is about an efficiency and endurance competition for vehicles powered by alternative energy sources. The objective is to drive the distance using the least amount of energy. And each vehicle must be designed and built by groups of students from education centers. The completion is located in the Cartagena Racetrack [2].

The Ánibal vehicle is an experimental solar car developed in the Technical University of Cartagena (UPCT) by students. Its aerodynamic body is made of carbon fibber and some parts are made of aluminium, titanium and wood. And for its movement uses two wheels in the front for the direction and a back wheel for traction.

Fig. 1 Ánibal Solar Car

The importance of energetic modelling is based on that it is possible to determine the energy generated by the photovoltaic system, and power consumption of the motor system. And because of the uneven ground in different places and variation of irradiation by month and place, you need to have a method to predict the performance of the whole system of solar vehicle.

2. Modelling Photovoltaic Collection System

The Ánibal’s solar collection system consists in three photovoltaic modules in series, a power regulator and two 12 volts batteries. The following table describes the
parameters of the photovoltaic module in standard conditions:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Descriptions</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isc (A)</td>
<td>Short circuit current</td>
<td>8.61</td>
</tr>
<tr>
<td>Voc (V)</td>
<td>Open circuit voltage</td>
<td>20.1</td>
</tr>
<tr>
<td>Pmax (W)</td>
<td>Maximum power Wp</td>
<td>130</td>
</tr>
<tr>
<td>NOCT (°C)</td>
<td>Operating Cell Temperature</td>
<td>47</td>
</tr>
<tr>
<td>Ns (Cells)</td>
<td># cells in series</td>
<td>32</td>
</tr>
<tr>
<td>Np (Cells)</td>
<td>Voltage variation with temperature</td>
<td>1</td>
</tr>
</tbody>
</table>

For the Cartagena Racetrack, it will be climatic data [3]:

**Irradiation in the horizontal plane Cartagena Racetrack**

<table>
<thead>
<tr>
<th>Month</th>
<th>Hh (Wh/m²/day)</th>
<th>TD (°C)</th>
<th>HPS (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct</td>
<td>3,990</td>
<td>22.2</td>
<td>3.99</td>
</tr>
<tr>
<td>Year average</td>
<td>5,150</td>
<td>20.4</td>
<td>5.14</td>
</tr>
</tbody>
</table>

Hh: Irradiation in the horizontal plane (Wh/m²/day)
TD: Average Temperature Day (°C)
HPS: Solar peak hour, hour of sunlight equivalent per day (hr)

\[ HPS = \frac{Hh(\text{Wh/m}^2)}{G_{\text{CEM}}(\text{W/m}^2]} \]  

With the HPS, the irradiance maximum (Gmax) and its time can be calculated the range of time when there is useful sunlight. The table 3 shows that for a HPS in October of 5.38 hours and with a Gmax of 575 W/m², it will have a range of time between 9:11AM and 2:33PM (the different point and irradiance data are in the Table 4). And the average of these irradiances data (Gm) is the representative irradiance for October.

<table>
<thead>
<tr>
<th>Month</th>
<th>HSP (hrs)</th>
<th>Gmax (W/m²)</th>
<th>Hour Gmax</th>
<th>Time range</th>
<th>Gm (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct</td>
<td>5.38</td>
<td>575</td>
<td>11:52</td>
<td>9:11AM-2:33PM</td>
<td>521.8</td>
</tr>
</tbody>
</table>

For estimations of photovoltaic module performance in operative conditions, it will be used the next equations for its characterization [4]:

\[ I_{SCCO} = I_{SC} \times \frac{Gm}{G_{CEM}} = A \]  

The power of the PV module (130 Wp) varies from 52 Wp as minimum to 70 Wp as maximum in operative conditions.

The figure 2 shows how can be estimated the solar energy available between ranges of time in Cartagena Racetrack for October. With the equation of the Irradiance/Time curve and doing a defined integration (Equation 8) with the time period that the solar car competes, it can be calculated.

\[ H_{h(\text{AT})} = \int_{T_1}^{T_2} (0X^4 - 0.066X^3 + 0.575X^2 + 36.53X + 21.06) \, dx \]  

And with next equation, we can estimate the electric energy that the PV system can generate:

\[ E_{FV} = \frac{PR \times H_{h(\text{AT})}}{G_{CEM}} \times P_{max} = Wh \]
3. Kinematic and dynamic model of Ánibal solar vehicle

For the propulsion modelling of the Anibal vehicle we could be divided in two parts. In one part we have the electric part, and the other the mechanic part.

The electric modelling part is based on a DC motor with independent exciting. The Figure 3 shows the equivalent circuit of this motor, so the operation of electric part is given as [5]:

\[ U_i(t) = \delta \] (10)

If the Ohm’s law is applied:

\[ U_i(t) = I_i R_1 + \] (11)

is given as [5]:

\[ \delta \] (12)

All velocity terms that shows in the text should be expressed in function of the linear velocity of the car however \( \omega(t) \) is the angular velocity of the wheel. The next equations shows how change the angular velocity to linear velocity:

\[ \omega(t) = \frac{d \theta(t)}{dt} \] (13)

If the equation (12) and (13) is substituted in equation (11), finally we have the next expression for the linear velocity:

\[ U_i(t) = I_i R_1 + \frac{d \theta(t)}{dt} + K_\delta \] (14)

For finalize the electric part, the electric torque is defined as [5]:

\[ T_{\text{me}}(t) = K_\delta \phi I_i(t) \] (15)

And the magnetic flow is given as:

\[ \phi = K_F I_2 \] (16)

If the equation (16) is substituted in equation (15), and we should remember that \( I_e \) is constant so the electric torque is given as:

\[ T_{\text{me}}(t) = K_F I_2 \] (17)

And now the mechanic part analysis will be developed. The Figure 4 shows the diagram of mechanical part.

\[ \omega_2(t) = G(t) \omega_1(t) \] (18)

\[ T_2 = T_1 G(t) \] (19)

This information will be utility in the following mathematical and physical definitions. So in our model the resistant torque is given as:

\[ T_r(t) = K_1 \frac{d \theta(t)}{dt} + K_2 \omega_1(t) + K_3 f(\omega) + K_4 (\omega_2(t))^2 \] (20)

This equation is composed of 4 terms. Where \( K1 \) is composed for 2 terms and represents the torque necessary for the acceleration of the car; the first term is given as [6]:

\[ T_{\text{ax, acc, car}}(t) = f \cdot \frac{d \theta(t)}{dt} = f \cdot \frac{\omega(t)}{e} \cdot \frac{d \theta(t)}{dt} \] (21)

The equation 21 represents the torque necessary for acceleration of the axle of the motor.

The other term of K1 is given as:

\[ T_{\text{ax, acc, car}}(t) = F_{\text{act, acc, car}}(t) \cdot d \] (22)

And the effort necessary for acceleration car is given as [7]:

\[ F_{\text{act, acc, car}}(t) = ma(t) = m \frac{\omega(t)}{d} \] (23)

Substituting the equation (23) in equation (22):
Finally \( K_1 \) is given as:

\[
T_{ac}^1(\tau) = m \cdot \frac{d^2 \varphi(t)}{dt^2} \cdot d \tag{24}
\]

\( T_{ac}^1(\tau) \) is the torque necessary for the speed. \( K_2 \) is given as [6]:

\[
T_{speed}^2(\tau) = B \cdot \omega(\tau) \tag{25}
\]

If the equation (26) is changed in function of the linear speed it is given as:

\[
T_{speed}^2(\tau) = B \cdot V^2(\tau) \cdot \frac{1}{2\pi} \cdot 60 \tag{27}
\]

Now \( K_3 \) will be explained as the term that represents the necessary torque for overcome the friction effort and the effort of weight. The Figure 5 shows these efforts.

\[
T_{fr}(\tau) = (\mu mg \cos(\alpha(t)) + m g \sin(\alpha(t)))d \tag{28}
\]

The equation (28) represents the torque in wheel, if this torque is change from wheel to the axe of the motor; it obtains the equation (29):

\[
T'_{fr}(\tau) = \frac{\mu mg \cos(\alpha(t)) + m g \sin(\alpha(t))}{\frac{1}{2\pi} \cdot 60} \tag{29}
\]

Finally \( K_4 \) represents the energy missed for aerodynamic friction. The torque of this energy missed is given as [8]:

\[
T_{aero}(\tau) = \frac{1}{2} \rho A C_d(V^2(\tau))^2 \cdot d \tag{30}
\]

The equation (30) represents the torque in wheel, if this torque is change from wheel to the axe of the motor; it obtains the equation (22):

\[
T'_{aero}(\tau) = \frac{1}{2} \rho A C_d(V^2(\tau))^2 \cdot \frac{1}{\frac{1}{2\pi} \cdot 60} \tag{31}
\]

This model has been implemented and simulated in Matlab and Simulink. The Figure 6 and 7 shows the principal screen of the program created. In the figure 6, the squares red and blue contains the modelling for the motor DC. And the green square in the figure 7, contains the equations for modelling all the terms (\( K_1, K_2, K_3 \) and \( K_4 \)) that affects the vehicle for calculation of the velocity.

The next figures are the result of the modeling simulation. The figure 8 represents the Voltage Vs. Time. And the figure 9 represents the Intensity Vs. Time.
Model Validation

To perform a more accurate analysis and get a real understanding of the behavior and performance of the solar car, acquiring experimental data is needed.

For that purpose, a monitoring system has been developed. It is composed of a series of sensors, and a real-time embedded controller. The sensors measure the electrical parameters of the vehicle (DC current and voltage of the motor system, DC current and voltage from the PV generator), and other variables like the speed and position. The embedded controller works like a data logger, which collects sensor signals, makes a small processing in the progress, and then stores them for later use.

This data allows us to confirm that the system behaves according to the design besides allowing us to make improvements and adjustments based on these experimental data, which results in a continuous improvement process.

Also, the acquired data allows us to test and improve the Solar Car mathematical model. A well-tuned model is an important tool to have reliable simulations. Through simulations, we can develop the upgrades at low-cost and on a short time.

The real-time embedded controller is Compact RIO from National Instrument. Voltage values of the battery bank and PV generator could be measured directly by the Compact RIO thanks to NI-9221, a module that can measure analog voltage up to 60V. Current values of the system are also measured by the module.

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Glossary:

- $U_i(t)$: drop voltage in induced (V)
- $\Delta U_r(t)$: drop voltage in resistance (V)
- $\Delta U_l(t)$: drop voltage in inductance (V)
- $U_m(t)$: drop voltage in motor (V)
- $I_i$: current in the induced (A)
- $R_i$: resistance of induced (Ω)
- $L_i$: inductance of induced (H)
- $\omega(t)$: angular speed in the axle (rad/s)
- $n(t)$: angular speed in the axle (rpm)
- $v(t)$: linear speed of the car (m/s)
- $r$: spoke of the wheel (m)
- $G(t)$: transmission ratio.
- $\tau_m(t) = \tau_a(t)$: torque in the axle of motor
- $\phi$: electromagnetic flow (Wb)
- $K_F$: constant (V/rpm)
- $I_e$: excitation current (A) (Constant)
- $\rho$: air density (Kg/m³)
- $Cd$: lift coefficient (No unit)

- $g$: gravity (m/s²)
- $\mu$: friction coefficient (No unit)
- $t$: time (s)
- $m$: total mass (Kg)
- $a$: acceleration of the car (m/s²)
- $J$: rotor inertia (Kgm²)
- $T_{rot}(t)$: torque necessary of acceleration of the motor (Nm)
- $T_a(t)$: total torque necessary of acceleration (Nm)
- $T_{speed}(t)$: torque required for speed (Nm)
- $B$: constant of viscosity terms of the motor (Nm/rpm)
- $t_{fr}(t)$: torque necessary because of weight and friction effort in wheel (Nm)
- $t_{air}(t)$: torque necessary because of friction effort with air in wheel (Nm)
- $T_{fr}(t)$: torque necessary because of weight and friction effort in axle of the motor (Nm)
- $T_{air}(t)$: torque necessary because of friction effort with air in axle of the motor (Nm)

- $\alpha$: angle of inclination (°)

- $\tau_{rot}(t)$: torque necessary of acceleration of the wheel (Nm)
- $J$: rotor inertia (Kgm²)
- $T_{rot}(t)$: angular speed in the wheel (rad/s)
- $d$: diameter of the wheel (m)
- $T_{acc}(t)$: torque necessary of acceleration of the car (Nm)
- $T_{tot}(t)$: effort necessary of acceleration of the car (N)
- $m$: total mass (Kg)
- $a$: acceleration of the car (m/s²)
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- $T_{fr}(t)$: torque necessary because of weight and friction effort in wheel (Nm)
- $g$: gravity (m/s²)
were measured with Hall Effect sensors. The current sensors output were connected to a NI-9201, an 8-channel analog input module.

All data is extracted from Compact RIO by FTP connection. A Matlab script makes this connection, download the data files and process them. An automatic report is generated by the script, which is well-explained and can be interpreted by any member of the team.

In the next figure, measured data of voltage and current data are shown as a result of the monitoring system.

![Figure 10](https://example.com/figure10)

5. Conclusion

The energetic modelling of a solar vehicle provides a general idea of what actually will be its functioning. It also provides information needed for sizing the drive system for the vehicle to move efficiently, and photovoltaic system uptake. Thus allowing knowledge of the energy that can be consumed and produced by location, month and length of time that the solar car driving.

Comparing the modelling result with the experimental result, we have different curves of voltage and intensity. Where the modelling curves represents the ideal behaviour for the vehicle in the track. With this reference we can make changes for the energy optimisation.

When there are situations where streets or roads have slopes, this energetic modelling is useful because we can estimate the energy consumption. Finally, energetic modelling can be used to program strategy for competitions such as the Murcia Solar Race, because we take in account not only considerations about the vehicle also race track considerations, and this is the differentiation with others vehicle simulator.

References

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