Abstract. Two energy hybrid filter systems were considered which reduce level of higher mains current harmonics. An advantage of such system is low demanded active filter power because in complements only passive filter operation. The first considered system has inverter switched on in series with shunt passive filter. Through optimization of control parameters it is able to eliminate specific higher mains currents harmonics left by passive filter in a point by point way. The other tested system includes two inverters which are switched on to the system in series and shunt way. Their operation is to reduce higher mains current harmonics induced by higher load currents harmonics in a continuous way. The simulations carried out proved correct operation of the system.

Key words

Hybrid filters, power electronics, current harmonics, simulation, optimization.

1. Introduction

Currents drawn from the mains by nonlinear receivers, for example for rectifier contain high level of higher harmonics. It is a cause of big problems about power grid. Passive filters are usually used to reduce level of higher harmonic in the mains. There are possible various passive filter systems which reduce individual harmonics that are present in the mains by making use of resonance effect. In addition, active filters are used in order to reduce the other harmonics which are not reduced by applied passive filters. In order to do that the applications of one or two inverters of proper control and proper-switch on into the system method was considered. An advantage of such hybrid use of active and passive filters is above all to reduce demented inverter power because its role is complementing passive filter operation only. It reduces only these harmonic which were left after passive filter operation. There are possible different active and passive filter systems to work with one another. There will be two energy hybrid filter systems considered in this article.
We assumed that $[V] = [V_1; V_2; V_3]$ inverter voltages can be written:

$$[V] = K \cdot [I_{Sh}] + D \cdot [I_{Lh}]$$

where $[I_{Sh}]$ and $[I_{Lh}]$ are higher mains current harmonics $[I_s]$ and load $[I_l]$. $[I_3]$ and $[I_2]$ higher current harmonics signed by $[I_{Sh}]$ and $[I_{Lh}]$ should be calculated. Then mains voltage $[E_s] = 0$, because we assume that mains voltages have not higher harmonics. We can write a note voltage formula $[U] = [U_1, U_2, U_3]$ from (Millman) voltage between notes theorem:

$$[V] \cdot \frac{1}{Z_F} - [I_{Lh}] = \frac{1}{Z_S} \cdot [I_{Sh}]$$

After the formula (5) substituted here we have:

$$(Z_S + Z_F + K)[I_{Sh}] = (Z_F - D)[I_{Lh}]$$

Two methods of reducing $[I_{Sh}]$ mains current harmonic result from the above formula. It can be attained by increasing $K$ – amplification value. Limitation on this increase is, however stability of this system [1]. On the other hand we should guarantee that, for specified pulsation $\omega_a$:

$$(Z_F - D)|_{\omega = \omega_a} \rightarrow 0$$

the aim of reducing mains current harmonics for these pulsations can be also attained. The equation (7) requires to separate higher harmonics from $[I_s]$ – mains currents and $[I_l]$ – load currents. In can be attained with the use of the digital filters. After filters applied and after equations (7) multiplied by Clark matrix from (1), we have:

$$(Z_S + Z_F + K \cdot G_s) \left[ \begin{array}{c} I_{Sh} \\ I_{Lh} \end{array} \right] = (Z_F - D) \left[ \begin{array}{c} I_s \\ I_l \end{array} \right]$$

Operation of filters is signed here by $G_s$ for mains currents and by $G_h$ for individual load current harmonics. In general, $G_s$ filter should be high – pass filter, $G_h$ – filters should pass only $(h \cdot \omega_a)$, where $\omega_a$ is fundamental pulsation. Harmonic sign is considered here which indicates a direction in which this harmonic rotates for three – phases system in the figure 1a. A “separate()” procedure was worked out to separate higher harmonics from $I_s$ - mains currents as well as to obtain specific $h$ – harmonics from 3 phases of $I_l$ - load current [4].

The figure 3 shows a method in which it works. Its operation is to bring currents from $(\alpha, \beta)$ – system to $(d, q)$ system rotating at $\omega_a$ – speed for $G_s$ – filter or at $(h \cdot \omega_a)$ - speed for $G_h$ – filter. Pulsations that interests as become zero – pulsations in this way. Next a high – pass filter for $G_s$ – filter are used. $G_h$ – high – pass filters releases frequencies which are higher than rather low set value in $(d, q)$ system.

![Figure 2. Three-phase hybrid filter system with 7-harmonic passive filter (we assume a system symmetry)](https://doi.org/10.24084/repqj10.621)
In corresponds to releasing pulsations higher than fundamental one in \((a, \beta)\) system. Whereas \(G_h\) – low – pass filter released zero pulsation which is precisely \((h, \alpha_h)\) pulsation in \((d, q)\) system. Filtering operation is carried out independently for \(d\) and \(q\) components of current \((10)\). After making this operation we return to \((11)\): We will work with \((a, \beta)\) coordinates. According to the formula \((9)\), the result is also multiplied by \(Z\) – filter impedance for \((h, \alpha_h)\) pulsation – for \(G_h\) – filter. The result is multiplied by \(Z = 1\) to apply the same procedure for \(G_i\) – filter. The presented method of filtering can be written by formulas for \(\alpha\) - set pulsations. The applied filters can be written as their transmittances:

\[
H(s) = \frac{B(s)}{A(s)} = \frac{Y(s)}{X(s)} \Rightarrow A(s) \cdot Y(s) = B(s) \cdot X(s) \quad (10)
\]

Filtering in made independently for \(d\) and \(q\) – components of signal across two the same parameter \((H(s)\) - transmittance) filter. The result of this procedure operation can be, however, written with the use of one formula. A fundamental operation is after – time differentiation in the formula \((10)\), which corresponds to multiplication by \(s\) – Laplace variable:

\[
\begin{align*}
&\begin{bmatrix} i_d \\ i_q \end{bmatrix} = d \begin{bmatrix} i_d \\ i_q \end{bmatrix} = d \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \\
&= \omega \begin{bmatrix} -\sin \omega t & \cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}
\end{align*}
\]

\[
(11)
\]

After making this operation we return to \((a, \beta)\) coordinates through multiplying by \(P.C^T\) - matrix. As a result of this operation we have \((i = \sqrt{-1})\):

\[
\begin{align*}
&\begin{bmatrix} i_d \\ i_q \end{bmatrix} = s \begin{bmatrix} \alpha & \beta \\ -\alpha & \beta \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}
\end{align*}
\]

\[
(12)
\]

In this formula \(\alpha\) is value of harmonic which we want to remove (at high – pass filter) or leave (at low – pass filter). \(S^\alpha\) – Laplace variable powers are in the formula \((10)\) which corresponds to multiply by proper matrix power from \((12)\):

\[
s^\alpha \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} s & \alpha \\ -\alpha & s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} -\omega_i + s & 0 \\ 0 & \omega_i + s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \cdot \text{inv}(V)
\]

\[
(13)
\]

where \(V = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}\).

After \((13)\) substituted, \(A(s)\) and \(B(s)\) – polynomials from \((10)\) become matrixes, that is

\[
Y(s) = \begin{bmatrix} A \left( \begin{bmatrix} s & \omega \end{bmatrix} \right) \right)^{-1} \cdot B \left( \begin{bmatrix} s & \omega \end{bmatrix} \right) \cdot X(s) = G(s) \cdot X(s)
\]

\[
(14)
\]

Digital filters are applied in program, therefore the above substitution should be used for \(z\) – variable rather than for \(s\) – Laplace variable, tat is we should used the formula \(e = e^{z^k}\), where \(dt\) is integration step (timing sample). After considering \((13)\) we have:

\[
z \equiv V \cdot \begin{bmatrix} \exp((-\omega_i + s) \cdot dt) & 0 \\ 0 & \exp((\omega_i + s) \cdot dt) \end{bmatrix} \cdot \text{inv}(V)
\]

\[
(15)
\]

A and \(B\) polynomial factors from \((10)\) of digital filters should be multiplied by \(z^{-k}\), \(k = 1, 2, ...\) following the formula \((15)\) in corresponds, at transformations described, to multiply by:

\[
z^{-k} \equiv V \cdot \begin{bmatrix} \exp(-k \cdot (-\omega_i + s) \cdot dt) & 0 \\ 0 & \exp(-k \cdot (\omega_i + s) \cdot dt) \end{bmatrix} \cdot \text{inv}(V)
\]

\[
(16)
\]
voltages. In order to do that the mains voltages are determined in \((d, q)\) co-ordinate system rotating at \(\omega_0\) - synchronous speed and low – pass filter is used. By having a positive – sequence voltage separated it is brought to \((\alpha, \beta)\) system that is \((eal, ebe)\) and \(R_{50}\) – angle of rotating of this positive – sequence voltage is determined [5]. In order to that, sine and cosine of this angle are calculated by using \((eal, ebe)\) voltages and next these values are passed independently through \((bf60, af60)\) low – pass filter. This filter passes frequencies up to 60Hz. Influence of higher harmonics on \(R_{50}\) – angle (directly on sine and cosine of the angle) is avoided in this way which can be calculated next from so sine and cosine values prepared by using are tangent function [7].

In the \(R_{50}\) – angle is exactly known it is used to hold \(U_{dc}\) – capacitor voltage that powers inverter at set level \(U_{dc0}\). Therefore, you can see higher level of higher mains current harmonics left.

Difficult operating conditions occur at low – pulsation loading fluctuations. Loading changes is determined by frequency \(R_o=30\ \Omega\) to \(R_o=5\ \Omega\). Cycle of changes is determined by frequency \(\Delta f=10\ \text{Hz}\). \(I_L\) – loading current oscillates from 12A to 25A on account of the system inductances. Difficulties are involved with \((f_0 \pm f)\) – frequencies that are present in mains currents at the time, where \(f_0\) is mains frequency [8]. According to the figure 3, 4 these frequencies occur in the “separate ()” procedure \((d, q)\) – system as \((\pm f)\) and should be stopped by \((bg50, ag50)\) filter. They should not be subject to damping by the considered system in this way. However, we can expect high values of necessary \([V]\) – inverter output voltages from the formula (5). It can cause the need for boosting planned system power.

As shown in the figure 10, inverter voltage, needed for removing higher mains current harmonics, can even reach 200V. It is due to low receiver resistance \(R_o=5\ \Omega\). It is not attainable at set inverter – capacitor voltage \(U_{dc0}=100V\). Inverter reduces proportionally the both components of this voltage so that its module equals \(U_{dc0}\).
3. Frequency characteristics of the energy hybrid filter system from the figure 2

The formula (9) in the domain of frequency should be carried out to obtain of frequency characteristics of the hybrid filter system from the figure 2. This formula is brought to diagonal from with the use of $C$ – matrix. This matrix separates from $(\alpha, \beta)$ components of currents their positive and negative – sequence voltage. These sequences in contrast with $(\alpha, \beta)$ components are not interdependent on each other, and $H_C$ – transmittance matrix is diagonal matrix.

$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \sqrt{2} \cdot V^{-1}$

$$H(j\omega) = \frac{I_{S \text{ compatible}}}{I_{S \text{ opposite}}} = \frac{C \cdot H(j\omega) \cdot C^{-1}}{H_C(j\omega)}$$

(17)

$H_C(j\omega)$ – absolute values of matrix elements are shown in the figure 11.

4. (by – pass) parallel and series two – inverter emergency hybrid filter

$U_A$ and $U_Z$ voltages should be calculated. These are voltages of active filter star points one of bridge – rectifier loading nodes respectively. $V_f$ – passive filter phase voltage should be calculated In advance to calculate $U_A$ – voltage. Current flowing across $R_p$ – resistance of each passive filter is calculated by knowing filter phases currents to be difference of mains currents $x(1:3)$ and currents across bridge – rectifier input $x(4:6)$, as well as currents flowing across filter inductance [4]. This resistance voltage drop along with filter potential across $C_p$ – capacitor produces filter phase voltage. $X$ – State variables are used here which are currents flowing across inductances and capacitor charges. While electrics circuit of mains phases and shunt inverter phases is used to calculate $U_Z$ by having potentials across $C_p$ – capacitors of inverter which are carried on side of filter circuits by measuring transformers [8]. From the condition it follows that a sum of mains current derivatives equals zero because there is not a circuit for mains currents zero component. Next $U_Z$ – voltage value is calculated by using $U_Z$ – voltage value in a similar way. By making use of $U_{A,j}$ – circuit, passive filers, inverter, $L_p$, $R_p$ and bridge input, $U_{A,j}$ – value is calculated so that a sum of current derivatives $x(4:6)$, flowing in the bridge will equal zero. The following state variables were assumed: $x(1:3)$ – mains currents, $x(1:3) = [I_{L_a,1}, I_{L_b,1}, I_{L_c,1}]$, $x(4:6) = [I_{L_a,2}, I_{L_b,2}, I_{L_c,2}]$, $x(7:9) = [I_{C_a,1}, I_{C_b,1}, I_{C_c,1}]$, $x(10:12) = [q_{C_a,1}, q_{C_b,1}, q_{C_c,1}]$, the same way $x(13:18)$ for by – phase of passive filter, $x(19:24)$
for c – phases of filter, \( x(25:27) = [i_{fja}, i_{fib}, i_{fic}], \) \( x(28:30) = [q_{Cfja}, q_{Cfjb}, q_{Cfjc}] \) for a, b, c – phases of series inverter output, the same way \( x(31:36) \) for shunt inverter, load current \( x(37) = I_s, x(38) C_{de}, x(39) \) capacitor charge, \( x(19) q_{Cde} \) – charge of common inverter capacitor. 

\[ V_{fja}, V_{fjb}, V_{fjc} \] – zero inverter voltages are calculated like \( U_{j1}, U_{j2} \) – voltages. A condition is also used here that a sum of current derivatives flowing out of inverters, that is \( x(25:27) \) for series inverter and \( x(31:33) \) for by – pass inverter will equal zero. Calculations of these four \( U_{j1}, U_{j2}, V_{fja}, V_{fjb} \) – voltages are made in a self – rectifying way which guarantees to set a sum of proper current derivatives at the end of time step to zero, even in the case when the sum of the currents does not equal (e.g. because of rounding errors) zero [2].

This is two – mode system showing one phase of main circuit for higher harmonics. Higher harmonic currents flowing into \( I_{jb} \) – bridge and voltages supplied by \( V_{ja} \) and \( V_{jb} \) – inverters are forced. Millman’s theorem gives \( U – node \) to – node voltage formula for this system.

\[
U = \frac{V_{g1} \cdot \frac{1}{Z_s} - V_{g2} \cdot \frac{1}{Z_F} - I_{jb}}{\frac{1}{Z_s} + \frac{1}{Z_F}} = V_{g1} - Z_s \cdot I_{sh} \quad (18)
\]

It is assumed that inverter control guarantees the following inverter output voltages:

\[
\begin{align*}
V_{g1} &= K_1 \cdot I_{sh} \\
V_{g2} &= K_2 \cdot V_F = K_2 \cdot Z_F \cdot (I_{sh} - I_{jb})
\end{align*} \quad (19)
\]

By inserting these voltages in the formula (18) we have:

\[
[(Z_s + Z_F) - K_1 - K_2 \cdot Z_F] \cdot I_{sh} = Z_F \cdot (1 - K_2) \cdot I_{jb} \quad (20)
\]

Unlike elimination by points of higher harmonics realized for hybrid filter system from the figure 2, according to the formula (9), for the considered system elimination of harmonics is realized in a continuous way after the formula (20). \( K_1 \) – high factor of negative – value should be assumed to guarantee as low \( I_{sb} \) higher mains currents harmonics value as possible. Whereas \( K_2 \) – factor should be close to 1. The system stability at set control should be also guaranteed. It is also essential that \( U_{dc} \) – potential across \( C_{de} \) – capacitor of inverters will keep \( U_{dc0} \) – set value. It is possible to guarantee by switching on fundamental harmonics voltage, which is in mains fundamental harmonics current phase, into series inverter output. Maximal value of this voltage should be proportional (PI – controller) to error \( \text{err} = U_{dc0} - U_{dc} \) of inverter – potential across a capacitor. However this method turned out to be small – stable.

5. Conclusion

The principles of two energy hybrid filter systems were presented. The first system from the figure 2 included passive filter which was switched on as a shunt. Inverter was switched on to this filter in series, controlled by the formula (9). This system eliminated higher harmonics in a point by point way. The amplitude diagram of its transmittance proves that, shown in the figure 9. The other system included passive filter also switched on in a shut way and also inverter switched on to this passive filter in series. However, in addiction the second inverter was switched on to the system in series way. According to the formula (20), such inverter control was to be worked out in order to obtain as high reduction of higher mains current harmonic as possible. This control was to select \( K_1 \) and \( K_2 \) – factors. So we can name it as a continuous control. It was only applied passive filter that reduced its characteristics pulsations in a point by point way. On the basis of the obtained courses during simulation we can recognize the both energy hybrid filter system to operate correctly. However, less cost of the first system attracts its attention which had very good results of filtering with the use of extended control system. With the use of it we can eliminate selected higher mains current harmonics in an easy way.

References