Comparative Analysis of a New Planetary Transmission
With Deformable Element Usable in RES

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Abstract. This paper presents the comparative analysis of a planetary transmission with deformable element in both running cases: as speed reducer, respectively as speed increaser.

Bellow is presented a synthesis algorithm to determine the constructive solution of the transmission using a practical application, and namely a micro-hydro power plant, consisting of a Kaplan turbine, the power transmission and an electrical generator.

Key words
Planetary chain-set transmission, deformable element, renewable energy systems RES, numerical simulation

1. Introduction

The use of power transmissions in the RES domain is needed in order to reduce or increase the angular speed of a motor or turbine. For instance, these transmissions are used as speed reducers in PV tracking systems (e.g. the worm drive), while in micro-hydropower plants or in wind turbines as speed increasers (e.g. the Henderson gearbox [3, 8, 5] in wind turbines and the belt transmission in micro hydropower plants [1]).

Generally, the transmissions used in RES are conventional ones, being characterized by large overall dimensions and/or low efficiencies. These disadvantages led to the necessity of implementing new planetary transmissions with reduced dimensions and higher efficiencies.

In order to reduce the transmissions’ radial dimensions, it is recommended to use the planetary transmissions. Taking this fact into account, this paper presents the comparative analysis of a planetary transmission with deformable element, proposed by the authors [4], in the two running cases: as speed increaser and as speed reducer. The comparison highlights the transmission properties that allow its implementation in RES.

The transmission proposed by the authors was obtained by applying a conceptual design algorithm. The planetary solutions with deformable element found in the technical literature are graphically systematized in Fig.1. By combining these three solving variants, several structures can be generated [4], from which the structure presented in Fig. 2 represents the final solution that meets the requirements.

Fig. 1. The planetary solutions with deformable element
Notations: def: deformable element, H-the carrier, 1-the satellite gear and 2-the sun gear

Fig. 2. The principle solution of the transmission

A synthesis algorithm used to determine the gears’ teeth numbers for a practically application is further presented. The algorithm requires adopting a transmission ratio for the speed reducer case, and, then, after reversing the power flow, keeping the same value for the speed increaser transmission ratio.

The example of the planetary transmission with i=10 (Fig. 2) is further presented in order to compare the properties of the two running cases. Taking into account this transmission ratio, the internal kinematical ratio can be determined using Willis’s relations [2, 9]:

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The relations between the transmission teeth numbers can be obtained using the internal kinematical ratio:

\[ i_0 = i_{14}^H \left( \frac{\omega_4}{\omega_3} \right) = \frac{z_4}{z_3} \cdot i_{oc} \cdot i_{os} , \]  

(4) 

The transmission with the closest kinematical ratio to the imposed value can be found imposing relations between the teeth numbers. For this purpose, the internal kinematical ratio can be determined using the following relation:

\[ i_0 = i_{14}^H \left( \frac{\omega_4}{\omega_3} \right) = \frac{z_4}{z_3} \cdot i_{oc} \cdot i_{os} , \]  

(5) 

where: 

- \( i_{oc} \) is the kinematical ratio for the sun gears and \( i_{os} \) is the kinematical ratio for the satellite gears.
- The relation between the transmission ratio and the internal kinematical ratio can be obtained for given values of \( i_{oc} \) and \( i_{os} \); afterwards, the teeth numbers of the transmission’s gears can be calculated.

As can be seen, when \( i_{os} = 1 \) the analysed transmission has 8 running cases:

- a) \( i_{oc} = 1; i_{aw} < 1; i_o < 1; i_{14}^H > 0 \); 
- b) \( i_{oc} = 1; i_{aw} > 1; i_o > 1; i_{14}^H < 0 \); 
- c) \( i_{oc} < 1; i_{aw} \leq 1; i_o < 1; i_{14}^H < 0 \); 
- d) \( i_{oc} < 1; i_{aw} > 1; i_o < 1; i_{14}^H < 0 \); 
- e) \( i_{oc} < 1; i_{aw} > 1; i_o > 1; i_{14}^H > 0 \); 
- f) \( i_{oc} > 1; i_{aw} < 1; i_o < 1; i_{14}^H < 0 \); 
- g) \( i_{oc} > 1; i_{aw} \leq 1; i_o > 1; i_{14}^H > 0 \); 
- h) \( i_{oc} > 1; i_{aw} > 1; i_o > 1; i_{14}^H < 0 \); 

a relation between the teeth numbers can be obtained determining \( i_0 \) (see Fig. 3), for different values of the \( i_{oc} \) and \( i_{os} \), and by considering \( z_4 = cT \).

Using the results from Fig. 4, a correlation between the teeth numbers of the transmission’s gears can be obtained, relation that fulfils the imposed requirements.

Taking relation 5 into account, two structural configurations were selected for \( i = 10 \) or \( i = -10 \):

- a) when \( i_0 = 0.9 \) and \( i_{oc} = 1.1 \) it results \( i_{os} = 1 \) (see Fig. 4,a) and, respectively, 
- b) when \( i_0 = 1.1 \) and \( i_{oc} = 1.1 \), it results that \( i_{os} = 1 \) (see Fig. 4,b).
2. Comparative Analysis

Similar to any other planetary transmission, the analyzed transmission can be used as speed reducer and as speed increaser. In the comparative analysis, the correlations presented above are taken into account for both running cases (speed increaser/ speed reducer).

The range in which the proposed transmission’s efficiency has values can be obtained considering the previous combinations between the teeth numbers and the internal efficiency ($\eta_0$) of a chain transmission [10]:

$$\eta_0 = \eta_{14}^\mu = \eta_{12}^\mu \cdot \eta_{34}^\mu = 0.81 \ldots 0.9604$$

The efficiency ($\eta$) of the two constructive solutions in both running cases can be obtained as follows:

- for the speed reducer:
  $$\eta_{\text{r}} = \eta_{14} = \frac{-\omega_{14}T_1}{\omega_{14}T_{14}} = \frac{-T_1}{T_{14}} \frac{1}{\omega_{14}/\omega_{14}} = \frac{-T_1/T_{14}}{1 - \omega_{14}/\omega_{14}} = \frac{1 - i_{0\text{r}}}{1 - \omega_{14}/\omega_{14}}$$
  where
  $$x = \text{sgn}(\omega_{14}T_1) = \text{sgn}\left(\frac{\omega_{14}T_1}{-\omega_{14}T_1}\right) = \text{sgn}\left(\frac{i_0}{1 - i_{0\text{r}}}\right);$$
  it results $\eta_{14}^4 = 0.69$ for $i_0 = 1.1$ ($i = -10$), respectively $\eta_{14}^4 = 0.75$ for $i_0 = 0.9$ ($i = 10$);

- for the speed increaser:
  $$\eta_{\text{i}} = \eta_{14} = \frac{-\omega_{14}T_1}{\omega_{14}T_{14}} = \frac{-T_1}{T_{14}} \frac{1}{\omega_{14}/\omega_{14}} = \frac{-T_1/T_{14}}{1 - \omega_{14}/\omega_{14}} = \frac{1 - i_{0\text{i}}}{1 - \omega_{14}/\omega_{14}}$$
  where $w = -x$;
  it results $\eta_{14}^4 = 0.55$ for $i_0 = 1.1$ ($i = -10$), respectively $\eta_{14}^4 = 0.65$ for $i_0 = 0.9$ ($i = 10$).

The variations of the efficiency for the speed reducer ($\eta_{\text{r}}$) and speed increaser ($\eta_{\text{i}}$), of the transmission ratio ($i$) and of the satellites internal kinematical ratio ($i_{0\text{s}}$) are presented in Figures 5 – 8, for $i_{0\text{r}}<1$ and $i_{0\text{i}}>1$. By analyzing the numerical simulations (Fig. 5-8), the following conclusions can be stated:

- the transmission has superior efficiencies when $i_{0\text{r}}<1$ (Fig. 5 and Fig. 6);
- for the same transmission ratio and kinematical ratio for the sun gears, the transmission has bigger overall dimensions when $i_{0\text{i}}>1$ (Fig. 7 and Fig. 8);
- different structural configurations can be obtained for the same transmission ratio (see Fig. 7 and Fig. 8).

3. Numerical Simulations

The analysis of the previous simulations (from Fig. 5 and Fig. 6) highlights the recommendation of using the transmission for relatively low values of the transmission ratio ($|i| < 25$).

The results of the numerical simulations from Fig. 7 and Fig. 8 can be compared based on the internal kinematical ratio ($i$) and considering that the internal efficiency of a chain transmission varies between 0.9 and 0.98.

From the simulations presented above (see Fig. 4 - Fig. 8) it outcomes that the variant with $i_{0\text{r}}<1$ is superior to the variant with $i_{0\text{i}}>1$; for this reason, the first solution will be

Fig. 5. The variations of the speed increaser/reducer efficiency ($\eta_{\text{i}},\eta_{\text{r}}$) and of the transmission ratio ($i$) vs. the internal transmission ratio ($i_{0\text{i}}$), when $i_{0\text{i}}<0$.

Fig. 6. The variations of the speed increaser/reducer efficiency ($\eta_{\text{i}},\eta_{\text{r}}$) and of the transmission ratio ($i$) vs. the internal transmission ratio ($i_{0\text{i}}$), when $i_{0\text{i}}>0$.

Fig. 7. The variations of the efficiency ($\eta$) and of the kinematical ratio for the satellite gears ($i_{0\text{s}}$) vs. the internal transmission ratio ($i_{0\text{i}}$), when $i_{0\text{i}}<1$.

Fig. 8. The variations of the efficiency ($\eta$) and of the kinematical ratio for the satellite gears ($i_{0\text{s}}$) vs. the internal transmission ratio ($i_{0\text{i}}$), when $i_{0\text{i}}>1$. 
considered, for which four running cases are taken into account:

a) \( i_{oc} = 1; i_{os} < 1 \); see Fig. 9;

b) \( i_{oc} < 1; i_{os} < 1 \); see Fig. 10,b;

c) \( i_{oc} < 1; i_{os} \geq 1 \); see Fig. 10,c.

d) \( i_{oc} > 1; i_{os} < 1 \); see Fig. 11.

From dynamic reasons, it is recommended that the fixed sun gear to be the largest in the 1 DOF planetary transmission. Therefore, the authors propose other combinations of teeth numbers, with comparative transmission efficiencies, in which the fixed gear is the largest (gear 4 from Fig. 2).

4. Case Study

Using the same computing algorithm, the authors propose an example of application for this transmission, namely speed increaser in a micro-hydropower plant equipped with a Kaplan turbine.

In order to increase the performances of the Kaplan assembly, the turbine has to work at lower speeds (300 rpm), while the generator has to work at higher speeds (1200 rpm) [7]. Therefore, a speed increaser with a multiplication ratio of 4 has to be placed between the turbine and the generator. For this case, the numerical simulations for the efficiency and kinematical ratios of the planetary speed increaser with deformable element are presented in Fig. 10 and Fig. 11.

Thus, the variation of the efficiencies and transmission ratio vs. the internal kinematical ratio are represented in Fig. 9,a, Fig. 10,a and Fig. 11,a; the variations of the kinematical ratio for the satellite gears vs. the internal kinematical ratio for different ratios between the sun gears teeth are represented in Fig. 9, b, Fig. 10, b, Fig. 11, b and c.

Applying the algorithm presented above (see Fig. 12) and using the data from Table 1, the following values for the gears teeth numbers and efficiency result for a kinematical ratio of \( i = 3.973; z1= 49; z2=22; z3=30; z4= 50 \) and \( \eta = 0.877 \); it was adopted \( z_4>z_1 \) from constructive reasons, and \( z_2>z_1 \) from dynamic considerations. Under these premises, the constructive variant of the planetary transmission is presented in Fig. 13, built using modern CAD/CAE software (Dassault Systemes CATIA, AutoDesk Inventor).

A possible implementation of the transmission from Fig. 13 in a micro-hydropower plant equipped with a Kaplan turbine is presented in Fig. 14.
- the transmission works better as speed reducer than speed increaser; however it is recommended to be used as speed increaser because it can run at relatively low transmission ratios (|\(i|<25\)). Therefore it is recommended to use it in the micro-hydro domain, where the transmission ratio is max. 6;
- in the same running conditions, the transmission with \(i_0<1\) has a better efficiency than in the case with \(i_0>1\);
- by fixing the largest gear due to dynamic considerations, transmissions with comparable efficiencies can be obtained modifying the proportions/ratios between the satellite gears teeth;

![Diagram](image)

Fig. 11. The variation of the efficiency and transmission ratio (a), kinematical ratio for the satellite gears (b) vs. internal kinematical ratio, when \(z_1>z_4\) (\(i_{oc}>1\)).

![Diagram](image)

Fig. 12. Application of the synthesis algorithm for \(i=4\) (see also Fig. 3 and 4)

![Diagram](image)

Fig. 13. The virtual model of the speed increaser prototype

The following conclusions can be stated based on the comparative analysis of the diagrams from Fig. 7 - Fig. 11:

### 5. Conclusions and Remarks

The following conclusions can be stated based on the comparative analysis of the diagrams from Fig. 7 - Fig. 11:

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<th>(z_1)</th>
<th>(z_2)</th>
<th>(z_3)</th>
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<th>(i_{os})</th>
<th>(i_{o})</th>
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<td>21</td>
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<td>18</td>
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Table 1. – The kinematical ratios and the efficiency for different combinations of teeth numbers
the efficiency ($\eta$) decreases with the increase of the transmission ratio ($i$);
- the planetary chain-set transmission allows a fine setting of the transmission ratio at reduced overall dimensions.

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**References**