Profit-Based Optimal Operation of a Head-Dependent Hydroelectric Power Station in the Bilateral Market

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Abstract. Deregulation and liberalization of electric power industry, among other things, has created new requirements for the market participants. The power system engineer, operator, and, in general, the market participant is being faced with requirements for which he does not have adequate training and the proper software tools. In this framework, among others, a pure hydro-generation company has to operate its hydro units, throughout the operating day, trying to fulfill the market clearing schedule or a bilateral contract, and modify the program in the intra-day energy markets if necessary (or more suitable) as real-time operation is getting closer. In this scenario the objective is to maximize the hydroelectric power plant profit from selling energy in the spot market or by means of bilateral contracts. In this paper the optimal operation of a head-dependent hydroelectric power station in bilateral market–short-term hourly hydro resource scheduling for energy– is obtained.

Keywords
Hydroelectric power generation, hydroelectric generators, optimization methods, electricity market.

1. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$K$</td>
<td>total number of hours in scheduling period</td>
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<tr>
<td>$f_k$</td>
<td>cost function (penalty or benefit function according to the fulfillment of the contract in each hour $k$)</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>power deviation relative to the contracted one for each hour $k$</td>
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<td>$U_k$</td>
<td>set of admissible decisions for plant units and reservoir; includes operation constraints, such as power limits and level limit constraints</td>
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<tr>
<td>$t_j$</td>
<td>tariff type $j$ at hour $k$</td>
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<tr>
<td>$P_N$</td>
<td>plant nominal power</td>
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<tr>
<td>$p_{ia}$</td>
<td>power output of unit $i$ in hour $k$</td>
</tr>
<tr>
<td>$D_k$</td>
<td>contracted generation requirement in hour $k$</td>
</tr>
<tr>
<td>$J$</td>
<td>total number of units in hydro resource</td>
</tr>
<tr>
<td>$l$</td>
<td>total number of curves in power station</td>
</tr>
<tr>
<td>$q_i$</td>
<td>draft (through the powerhouse) corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$q_{i}^{\text{max}}$</td>
<td>maximum draft corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$q_{i}^{\text{min}}$</td>
<td>minimum draft corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$p_{ij}$</td>
<td>power output of unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$p_{j}^{\text{max}}$</td>
<td>maximum generating capacity corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$p_{j}^{\text{min}}$</td>
<td>minimum generating capacity corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$p_{j}^{\text{max}}$</td>
<td>maximum generating capacity corresponding to unit $j$ (whatever the curve $i$)</td>
</tr>
<tr>
<td>$p_{j}^{\text{min}}$</td>
<td>minimum generating capacity corresponding to unit $j$ (whatever the curve $i$)</td>
</tr>
<tr>
<td>$P$</td>
<td>total power output generated by plant (power demand)</td>
</tr>
<tr>
<td>$Q$</td>
<td>total draft through all the committed units</td>
</tr>
<tr>
<td>$h_i$</td>
<td>head of curve $i$</td>
</tr>
<tr>
<td>$u_j$</td>
<td>decision variable for unit $j$</td>
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2. Introduction

The satisfaction of the demand for electric energy has been mainly achieved with hydro resources and thermal resources. Hydro resources particularly run-of-the river resources are considered to provide a clean and environmentally friendly energy option, while thermal resources particularly fossil fuel-based resources are considered to provide an environmentally aggressive...
energy option, but nevertheless still in nowadays a necessary option. The Portuguese fossil fuels energy dependence is among the highest in the European Union. Portugal does not have endogenous thermal resources, which has a negative influence on Portuguese economy. Moreover, the Portuguese greenhouse emissions are already out of Kyoto target and must be reduced in the near future. Hence, promoting efficiency improvements in the exploitation of the hydro resources is increasingly important, reducing the reliance on fossil fuels and decreasing greenhouse emissions, which are major contributors to climate change, we report our research concerning efficiency improvements applied on a case study based on one of the Portuguese hydro power plant, thus providing a higher profit for the generation companies.

A. Changes within the Electricity Industry

Electricity industry restructuring has received government priorities worldwide while restructuring policies are debated at all levels internationally. The preliminary experiences have shown that the establishment of electricity market is going to be specific to legislations, cultures, economy, and electricity operations and practices in participating nations [1]. Portugal is also moving towards a competitive electricity market with the presumption that the competition will result in technological progresses, better services, higher efficiency and enhanced reliability, as well as less costly delivery of electricity to customers. In a deregulated profit-based environment [1]-[3], such as the Norwegian case [4] or concerning Portugal and Spain given the Iberian electricity market, the optimal management of the water available in the reservoirs for power generation, without affecting future operation use, represents a major advantage for generation companies to face competitiveness given the economic stakes involved. The main goal in the profit-based hydro scheduling problem is to maximize the value of total hydroelectric generation throughout the time horizon, while satisfying all hydraulic constraints, aiming the most efficient and profitable use of the water [5]. Hence, the improvement of existing hydro scheduling models promoting a better exploitation efficiency of hydro resources is an important line of research [6], especially for head-dependent reservoirs in light of market conditions [7], [8]. The efficiency characterizes the conversion of the potential energy contained in the water discharged through the turbines into the gross hydro energy output [9]. The hydro generation characteristics are mainly assumed as linear or piecewise linear in hydro scheduling models, neglecting head variations. For long-term time horizons, the linearity assumption is reasonable, since errors introduced by this assumption are expected to be small compared to uncertainties with respect, for instance, to hydro inflow [10]. For a particular configuration of the hydro system, the linearity assumption may be acceptable or not for short-term time horizons depending on how important is the head variation over the time horizon. In hydro plants with a large storage capacity available, as it is the case in the Brazilian system for instance, head variation has negligible influence on power generation efficiency in the short-term [11], and the linearity assumption is acceptable. In hydro power plants with a small storage capacity available, also known as run-of-the-river hydro plants, the power generation efficiency can change significantly due to the non-linearity generation characteristics. For instance, in the Portuguese system there are several hydro power plants with small reservoirs. Hence, it is necessary to consider the head dependency characteristic – nonlinear dependence between the power generation, the water discharge and the head – in order to obtain more realistic and feasible results.

The electric utility deregulation and restructuring in Portugal has been implemented in a step-by-step way, and is now based on the existence of both Public Service Electric System (SEP) and Independent Electric System (SEI). The Non-binding Electric System (SENV) is part of SEI. The non-binding client is an individual or corporate body, the holder of an electric energy consumer installation, which has been authorized access to the SENV. The non-binding producer is the holder of a non-binding electric energy production license, by which it is authorized to carry out the activity of the production of electric energy within the ambit of the SENV. Concerning to the economic relationships between SEP and SENV, these are clearly regulated, and within the SENV, it can be done by physical bilateral contracts, contracts for small time period, contracts with guaranteed delivery or through offers system contracts [12]. By this mean, the electricity market liberalization process has introduced power generation concurrency as well as the possibility of the consumer (non-binding client) to choose which deliverer he wants (non-binding producer). This new scenario brings new problems in electric energy management. One of these new problems is the exploitation of hydro power plants, and is within the responsibility of the non-binding producer. It can be state that, for optimizing power generation efficiency for head-sensitive cascaded reservoirs, the problem solution requires the achievement of the powerhouse input-output characteristics, considering the non-linearity generation characteristics [13]-[15]. Thus, for deregulation applications, Short Term Hydro Scheduling (STHS) solution is very important as a decision support to elaborate a daily operation plan of the hydro resources in order to assess the available energy that will be delivered by the non-binding producer. This problem have not received great attention and not much has been published on this subject, largely due to its complexity, resulting from the accuracy (the result must be a realistic value of power for each unit and not the water discharge of plant) and the real-time needed in the problem solution. The published work is mainly concerning the dispatch of hydro generating units considering head dependence [16], [17].

B. Organization

The paper presents the main problem and its mathematical formulation, as well as the computational adopted method for solving it. After, some illustration results are presented and finally some conclusions are taken.
The paper is structured as follows: Section 3 provides the mathematical formulation of the profit-based optimal operation of a head-dependent hydroelectric power station in the bilateral market; Section 4 presents the mathematical formulation and the solution to obtain the power house I/O curves considering head dependency; Section 5 presents a case study, illustrating the numerical results; Section 6 provides conclusions.

3. Problem formulation

A non-binding producer, which has established a bilateral contract, must put power into the grid (assuming that there is technically feasibility) that the non-binding clients will consume in order to fulfill the contract. That contract constitutes, for the non-binding producer, the exploitation program that, by rule, consists of the power that he must deliver to the network, each hour, during a day. If he doesn’t fulfill the contract (by default or by excess) it will incur in costs or incomes associated with deviations. These deviations result from the difference between the contracted values and those recorded in practice, and are calculated based on nominal power $P_n$, which the producer has installed. In this case, a non-binding producer is responsible for a hydroelectric power plant whose exploitation he intends to manage optimally along the day (duration of the contract). So, the optimal exploitation problem of the plant includes the contracted load profile, the penalties for production deviations from the contracted profile and the constraints associated with the hydroelectric power plant. The resolution of this problem allows achieving the optimal production profile. In this case, the goal isn’t to meet the load profile, but to minimize costs and, if possible, to achieve production benefits. Thus, the formulation of the problem $(\mathcal{P})$ is the following:

$$\begin{align*}
(\mathcal{P}) & \quad \text{Min } \sum_{k=1}^{K} f_k(\delta_k) \\
\text{subject to } & \\
\delta_k & \in l_k
\end{align*}$$

The objective function of problem $(\mathcal{P})$ results from a sum of functions, a function for each hour $k$, and each function $f_k : \mathbb{R} \rightarrow \mathbb{R}$ is defined as follows:

$$f_k(\delta_k) = \begin{cases} 
-\delta_k - t_{\alpha} & \text{if } \delta_k < -\beta P_n \\
-\delta_k - t_{\alpha} & \text{if } -\beta P_n \leq \delta_k < -\alpha P_n \\
-\delta_k - t_{\beta} & \text{if } -\alpha P_n \leq \delta_k < \alpha P_n \\
0 & \text{if } \alpha P_n \leq \delta_k \leq \beta P_n \\
0 & \text{if } \beta P_n < \delta_k
\end{cases}$$

where

$$\beta > \alpha \text{ for } \beta \in \mathbb{R}^+ \text{ and } \alpha \in \mathbb{R}^+$$

and

$$\delta_k = \sum_{i=1}^{i} p_i - D_k$$

As mentioned, the objective function is a penalty or benefit function according to the fulfillment of the contract in each hour $k$. The penalty or benefit depends on the deviation and it’s always linear. The angular coefficients of each penalty are given by $t_{\beta}$, and obey the following relation: $t_{\beta} > t_{\alpha} > t_{\alpha}$. Thus, for deviations smaller than $\alpha \times 100\%$ we get benefit or penalty (according to the tariff $t_{\alpha}$); for negative deviations among $\alpha \times 100\%$ and $\beta \times 100\%$ the penalty is higher (according to the tariff $t_{\beta}$) and it’s still aggravated for negative deviations above $\beta \times 100\%$ (according to the tariff $t_{\beta}$); for positive deviations above $\alpha \times 100\%$ there is no benefit or penalty.

As shown, each of the partial functions $f_k$ is discontinuous, nonlinear and nonconvex. These properties of function $f_k$ raise difficulties to achieve the solution of the problem $(\mathcal{P})$ and require an optimization beyond the field of conventional nonlinear programming. The method used here to overcome this difficulty is an implicit enumeration method, in which all possible decisions are tested and the best decisions are then chosen. This ensures that the results are optimal and globally optimal. The disadvantage of this method comes from the requirement to work in a discretized space, requiring more memory and runtime. To avoid that the runtime makes the use of this method not viable, it requires a precise implementation of the algorithm that is based, essentially, in an efficient data structure. Thus, it is possible to reduce operations that, for being repetitive, lead to a significantly increased runtime. When solving the problem $(\mathcal{P})$, is essential to know, at each level and for each possible draft to turbine, the best combination of unities that corresponds to the maximum energetic efficiency (for a given head and draft) and the power allocated to each unit. This problem is a unit commitment problem in the hydro plants and will be presented in the next section.

4. Powerhouse I/O curves considering head dependency

The hydro generation model is either unit – or plant-based. For a more accurate approach, each individual unit in a plant is treated separately, which yields a hydro unit commitment problem. In this paper we adopt an aggregated plant concept, where units in a hydro plant are aggregated as one equivalent plant, but the unit commitment in the power plant can change, according to the head and the water flow to achieve optimal solution. The electric power generated is computed as a function of water flow, depending on hydro unit input/output (I/O) characteristic associated with the corresponding head. The dispatch of head dependent hydro units (set of characteristic curves, each one for a constant value of electric generated power, for each hydro power plant) incorporates water flow unit limits, unit generated power limits and the head dependency.
effect. In particularly, this problem assumes a great complexity when the units in a power plant are different from each other, mainly because some of them saw its capacity increased, and because the objective function is non-linear and non-convex. For these reasons the problem solution imposes an optimization out of conventional non-linear programming (increasing the runtime). The advantage of using the aggregated plant concept is that it can be done offline, reducing significantly the time required in the optimization process in hourly hydro resource scheduling, for energy.

C. Mathematical formulation

Given the imposed constraints, those required for each unit and those connected with all units, a proper unit commitment decision must be chosen and must be optimal from the economic benefit point of view. This problem involves, by one way, the statement of all possible decisions and the value associated with each of them, and by another way, the strategy analysis used to achieve the optimal solution. Thus, the problem formulation brings another problem, of mathematical programming, non-linear, described as follows.

Consider a hydro power plant with \( J \) units. Each unit is characterized by three variables: power, water flow and head. If one of these variables is kept constant – let be the head – each unit \( j \) is characterized by a set of curves. The number of curves \( I \) is as big as bigger are the discretization levels, assumed for the head.

Each curve \( i \) of unit \( j \) can be represented as a function of the generated power and the net head:

\[
q_i = f(p_j, h_i)
\]

with

\[
i = 1, \ldots, I \quad \text{and} \quad j = 1, \ldots, J
\]

The goodness of different possible decisions is made based on an established scale that characterizes each solution. This measurement scale is obtained from a function – objective function. The objective function that better fits the problem under analysis is the water flow through the turbines within the powerhouse (the water flow represents the operating cost).

Thus, expression (1) is a cost operation function, and the main problem to determine the dispatch of head dependent hydro units (power plant characteristic curves) is related to the optimal unit commitment problem, and can be presented as follows.

For a set of units within a hydro power plant, minimize the operating cost, according to:

- power demand – constraint connected with all units
- minimum and maximum generating capacity of each unit depending on head – constraint on individual curve
- minimum and maximum generating capacity of each unit independently on head – constraint on individual unit

So, the hydro unit commitment problem, for each curve \( i \), can be written as:

\[
\text{Min}_{u} \left( \sum_{j=1}^{N} q_j(p_j, h_i, u_j) \right)
\]

subject to:

\[
\sum_{j=1}^{J} p_j = P
\]

\[
p_j^{\min} (h_i) < p_j < p_j^{\max} (h_i) \cup p_j^{\min} < p_j < p_j^{\max}
\]

where:

\[
u_j \in \mathcal{L}_j \quad j = 1, \ldots, J
\]

Expression (2) represents the total value of water flow and indicates that for a specific value of generated power \( P \), with head \( h_i \), the water flow depends on the unit’s dispatch for the considered unit commitment. Expression (3) represents the generated power by the plant, for the considered unit commitment. Expression (4) is the result of considering the minimum and maximum generating capacity of unit \( j \) in curve \( i \), together with the minimum and maximum generating capacity of unit \( j \) whatever the curve is. The expression (5) represents the resource feasibility set.

D. Illustration results—without considering the elevation of the downstream head

As an illustration, we consider the case study of a small hydro power plant with six units, G1-G4 (identical units), G5 and G6. Each unit is characterized by eight curves, \( I = 8 \), and the relation between heads is given by \( h_{i+1} > h_i \) with \( i = 1, \ldots, 8 \). In this example, the problem solution allows to obtain eight characteristic curves for the power plant – the same number of curves that characterizes each unit, without considering the elevation of the downstream head with the water flow through powerhouse.

Fig. 1 and Fig. 2 show the characteristic curves of the hydro power plant, for constant values of head and for constant values of power, respectively.

![Fig. 1. Set of characteristic curves of the hydro power plant for constant head values.](https://doi.org/10.24084/repqj08.700)
For any value of power $P$ generated by the hydro power plant, and for the considered values of head $h$, the water flow $Q$ is minimum, defining the unit commitment ($Q$ is the total water flow through the committed units). In the case of unit commitment involving a combination of units, the level of power generation is different for each one of them – Fig. 1 shows the total values of power and water flows. Note that a discontinuity exists, near low power area, caused by the transitions between different unit commitments. This fact results from both the different characteristics of each unit and the generating capacity limits. Except for the critical area, the curves have a smooth and continuous evolution.

Fig. 2 shows the characteristic curves of the hydro power plant for a constant value of generated power. This figure shows the increase in water flow needed to generate the same value of power with the decrease in head. It can be seen that for some values of power, the unit commitment changes, according to the head and the water flow to achieve optimal solution. The critical area referred above can also be seen near low power values. Also, Fig. 2 shows the obtained different unit commitments with different colors. Each color represents a different combination of units. Note that is possible to obtain up to nine different commitments for the same generated power, up to five different commitments for the same water flow and up to ten different commitments for the same head.

Fig. 2. Set of characteristic curves of the hydro power plant for constant generated power values and the corresponding unit commitment according to the color map.

5. Illustration results

The numerical results of the problem’s solution ($\mathcal{P}$) is now presented for the case considered in previous section. In the resolution of this problem the goal is not to satisfy precisely the load profile, but to minimize costs and, if possible, to obtain production benefits. That is, with the available data (inflow to the reservoir every hour, initial and final reservoir levels) and considering all the problem constraints, the question that we intend to see answered is the following: what is the exploitation profile that allows to achieve this goal? The resolution of the problem answers optimally to this question, as we will illustrate below.

As mentioned in Section 3, the objective function of problem ($\mathcal{P}$) results from a sum of functions, a function for each hour $k$, and it is a function described in terms of more than one expression, depending on the value of the parameters $\alpha$ and $\beta$. In this illustration example the following values for parameters $\alpha = 0.05$ and $\beta = 0.15$ was considered.

Fig. 3 shows that the load is never exactly satisfied. By the contrary, there are marked differences, especially in the off-peak hours where the production is about four times lower than the contracted power. During full and peak hours the production is higher than the contracted power, except for a few hours (full hours). Note that, in terms of energy, its distribution in relation to the total energy of the contracted load diagram is the following: energy in off-peak hours, 27.33%; energy in full hours, 49.02%; energy in peak hours, 23.65%. From the total contracted energy, with the referred distribution, the plant is able to satisfy 92.11% of this energy, with the following distribution: 5.73% of the energy in off-peak hours, 58.27% of the energy in full hours, 36% of the energy in peak hours. We can immediately see that the exploitation optimization is not intended to meet the load, but to follow the objective of minimizing costs and obtaining benefits that can result from the bilateral contract. If we do the analysis in terms of percentage of each type of hourly tariff, then this conclusion is further supported. From the total contracted energy according to the hourly tariff, the plant meets 22.31% of the energy in off-peak hours, 109.48% of the energy in full hours and 121.33% of the energy in peak hours. There is an excess of production in peak hours and full hours, in order to obtain benefit at the expense of losses in off-peak hours.

Fig. 3. Optimal operation profile. The bar with three shades of gray represents the hourly tariff: light gray – off-peak hours tariff, intermediate grey – full hours tariff and dark gray – peak hours tariff.
6. Conclusions

This paper addresses the optimization problem of the exploitation of a head-dependent hydroelectric power station in the bilateral market. In this new scenario the solution of this problem was obtained. In particular, its formulation and its solution were illustrated.

First, the problem of unit allocation in hydro power plants was formulated and, subsequently, was solved. The solution of this problem has enabled to achieve the optimal units allocation. Thus, characteristic curves were obtained for the plant, which correspond to maximum energetic efficiency, and allows to know all the values of power that the plant can produce, which are the units that must be used, which inflow and what power level. These results are essential in solving the profit-based optimal hydro operation in a bilateral power market.

The obtained results allowed (1) to get the profile of the optimal exploitation for a specific bilateral contract, between a non-binding producer and a non-binding client, (2) to show how the optimal exploitation is done in the context of the restructuring, the new requirements and the new behaviors and (3) to show that the exploitation of a resource, in this new framework, obeys to different criteria from the usually used, which results in changes in how to operate the plant, always difficult to achieve and implement, because it is different from the traditional way.

References