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Abstract. According the characteristics of both steam and power was supplied in paper mill power plant, the optimal running model of the coal-fired power-heat system was researched in this paper. The power plant’s physical structure was researched, the superstructure was established and the boiler and turbine running model was analyzed. With the help of mathematical programming, each turbine unit optimal loads were obtained when the same power and steam were produced, moreover, the minimum coal cost were calculated with each boiler unit optimal load distribution. The case study of one paper mill showed that the optimization model was helpful and the coal consumption can decrease 4.7% aiding with the model.

Key words: optimal load distribution; mathematical programming; power-heat system; paper mill

1. Introduction

In the recent years, the production of paper mill increased rapidly with the development of economy in China. Meanwhile, the pressure of energy consumption cost also became heavier for most of paper mill managers because energy is the third place cost for one paper mill except fibre material cost and chemical cost. And as the energy conversion section of paper mill, power plant has a great influence on the mill wide energy efficiency, so how to save energy in power plant become an important issue for pulp and paper engineering researchers.

Nowadays, most power plant of paper mill in China is coal-fired power-heat system which supply power for paper machine and other instruments and mainly supply steam to paper machine dry section. And many paper mills have self governed power plant, most of them have more than one turbines and one boilers. For this kind of paper mill power plant, different turbine unit load and boiler unit load will lead to different coal consumption when producing the same power and steam. The turbine unit loads and the boiler unit load were very important because there is an optimal load distribution which will result in the minimum cost.

To get the minimum cost, mathematical programming often be used and mixed-integer linear programming (MILP) is a very general framework for capturing problems with both discrete decisions and continuous variables H.K. Sarimveis [1] used MILP model to optimize a paper mill energy system. M. Bojic [2] used MILP to optimize the cogeneration system of one paper mill, and Hui C. W. [3] optimized one Japanese petrochemical enterprise’s power plant with the help of MILP.

In this paper, mathematical programming was also used to get the optimal running model of the coal-fried power-heat system, and after optimization, the minimum cost turbine unit and boiler unit load distribution was obtained.

2. Boiler Model

The boiler model is a relationship between boiler efficiency and evaporation at different load. Because the demand fluctuation of steam in paper mill is big, the boiler load will change frequently, it bring some difficult for objective function and constraint function.

A. Objective function

The objective of boiler unit optimization is to get the minimum coal consumption when meeting the requirement of steam production, as in (1)

\[ \min B = \sum B_i \] (1)
Where \( B \) is total boiler coal consumption; \( B_i \) is \( i \)-th boiler coal consumption.

### B. Constraint Function

#### 1) Boiler characteristics constraint function
Boiler characteristics constraint function reveals the relationship between each boiler coal feeding to evaporation, as in (2) and (3)

\[
B = f(S_i) \quad (2)
\]

\[
B_i = a_0 y_i + a_1 S_i + a_2 S_i^2 + \ldots + a_n S_i^n \quad (3)
\]

Where \( B \) is boiler coal consumption, \( S_i \) is the steam outlet of boiler \( i \), \( B_i \) is boiler \( i \) coal consumption.

#### 2) Steam load constrain
Steam load constrain expresses the turbines requirement to boiler system steam production, as in (4)

\[
\sum_{i=1}^{n} S_i = S \quad (4)
\]

where \( S \) is the total steam production of boiler system.

#### 3) Boiler evaporation constrain
Boiler evaporation constrain means the range of each boiler operation, as in (5)

\[
(S_i)_{\text{min}} \leq S_i \leq (S_i)_{\text{max}} \quad (5)
\]

### 3. Turbine Model

The turbine model is to establish the relationship among steam feeding rate with power generation and steam extraction amount.

#### A. Objective function

Turbine objective function is to get the minimum steam feeding yet the power generation is still the same, as in (6)

\[
\min S = \sum_{i=1}^{n} S_i + \sum_{j=1}^{m} D_i \quad (6)
\]

where \( S \) is the total steam demand; \( \sum_{i=1}^{n} S_i \) is the sum of turbine \( n \) steam feeding; \( \sum_{j=1}^{m} D_i \) is the steam feeding sum of turbine \( m \) pressure and temperature reducing valve.

#### B. Constraint Function

##### 1) Power generation constraint
This constraint is to meet the power user’s requirement, as in (7)

\[
\sum_{i=1}^{n} N_i = P \quad (7)
\]

where \( N_i \) is generated power of turbine \( i \), \( P \) is total demand of power.

##### 2) External steam constrain

The external steam constraint is to meet the requirement of the balance of inlet and outlet steam, as in (8)

\[
\sum_{j=1}^{m} (S_{\text{in},j}) - \sum_{j=1}^{m} (S_{\text{out},j}) = S_{\text{load}} \quad (8)
\]

##### 3) Facility model constrain

Facility model constrain reveals the relationship among inlet steam, outlet steam and power, for extraction steam turbine, the facility model constrain as in (9)

\[
S_{\text{in}} = a_0 + a_1 N_i + a_2 S_{\text{in},1} + a_3 S_{\text{in},2} \quad (9)
\]

where \( S_{\text{in}} \) is steam inlet of turbine \( i \), \( N_i \) is the generated power of turbine \( i \), \( S_{\text{in},1} \) is the generated power of turbine \( i \), \( S_{\text{in},2} \) is the low pressure extracting steam amount.

##### 4) Low pressure cylinder minimum inlet steam constrain

Low pressure cylinder must keep a minimum steam flow rate to cool the machine, the constrain is as in (10)

\[
S_{\text{in}} - S_{\text{in},1} - S_{\text{in},2} - \sum_{j=1}^{m} S_j > (S_{\text{COND}})_{\text{min}} \quad (10)
\]

where \( S_j \) is the extracting steam sum of heating regenerator which usual gets through balance calculation.

##### 5) Maximum inlet constraint

The maximum inlet constraint is that the real inlet steam must not exceed the design value, as in (11)

\[
S_{\text{in}} \leq S_{\text{in},\text{max}} \quad (11)
\]

where \( S_{\text{in}} \) is the real inlet steam; \( S_{\text{in},\text{max}} \) is the design inlet steam value.

##### 6) Maximum generation power constraint

In the real operation, turbine can run over rated power, but the optimization must guarantee the safety and keep turbine running below rated power, as in (12)

\[
N_i = N_{\text{max}} \quad (12)
\]

Where \( N_i \) is the generated power of turbine \( i \), \( N_{\text{max}} \) is the rated generated power of turbine \( i \).

##### 7) Extracting steam constrain

To keep turbine working well, the first extracting steam amount and second extracting steam amount can’t exceed the set value, and the constraint is as in (13)

\[
S_{\text{in},1} \leq (S_{\text{in},1})_{\text{max}} \quad S_{\text{in},2} \leq (S_{\text{in},2})_{\text{max}} \quad (13)
\]
Where $S_{1i}$ is the medium pressure extracting steam amount of turbine $i$, $S_{2i}$ is the low pressure extracting steam amount of turbine $i$, $(S_{1i})_{\text{max}}$ is the medium pressure maximum extracting steam amount of turbine $i$, $(S_{2i})_{\text{max}}$ is the low pressure maximum extracting steam amount of turbine $i$.

8) Variable constraint

Variable constraint limit the range of variable fluctuation, as in (14)

$$S_{1i} \geq 0; S_{2i} \geq 0; N_i \geq 0$$  (14)

4. Case Research

Taking one paper mill power plant as the research case. There are four boilers and four turbines. Boiler #1, 2, 3 and turbine #1, 2, 3 share one steam main pipe and is a main pipe thermal system, and boiler #4 and turbine #4 use a separate main pipe and is a unit thermal system. And the turbines and boilers character is as in Table I and II.

| Table I.-Main capacity parameters of turbines |
|-----------------|-----------------|-----------------|
| **Items**       | **Unit**        | **Design capacity parameter** |
|                 | #1              | #2, 3, 4         |
| Rate power      | kw              | 50000            | 80000            |
| Steam pressure  | Mpa             | 12.3             | 12.3             |
| Medium pressure | Mpa             | 1.6              | 1.6              |
| Medium extracting steam pressure | t/h         | 30               | 50/50/90         |
| low pressure extracting steam pressure | Mpa        | 0.6              | 0.6              |
| low pressure extracting steam amount | t/h         | 145              | 195/195/300      |
| Exhaust pressure | Mpa            | 0.007            | 0.007            |

| Table II.-Main capacity parameter of boilers |
|-----------------|-----------------|-----------------|
| **Items**       | **Unit**        | **Design capacity parameters** |
|                 | #1              | #2, 3, 4         |
| Evaporation capacity | t/h        | 250              | 400              |
| Superheated steam pressure | Mpa    | 12.5             | 12.5             |
| Superheated steam temperature | °C    | 537              | 537              |
| feed-water temperature | °C        | 140              | 140              |

8) Variable constraint

Variable constraint limit the range of variable fluctuation, as in (14)

$$S_{1i} \geq 0; S_{2i} \geq 0; N_i \geq 0$$  (14)

4. Case Research

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<table>
<thead>
<tr>
<th>Items</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate power</td>
<td>kw</td>
</tr>
<tr>
<td>Steam pressure</td>
<td>Mpa</td>
</tr>
<tr>
<td>Medium pressure</td>
<td>MPa</td>
</tr>
<tr>
<td>Medium extracting steam pressure</td>
<td>t/h</td>
</tr>
<tr>
<td>low pressure extracting steam pressure</td>
<td>Mpa</td>
</tr>
<tr>
<td>low pressure extracting steam amount</td>
<td>t/h</td>
</tr>
<tr>
<td>Exhaust pressure</td>
<td>Mpa</td>
</tr>
</tbody>
</table>

Table III.- Turbine #4 regression model

<table>
<thead>
<tr>
<th><strong>Non-standard coefficient</strong></th>
<th><strong>Correlation coefficient</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Standard Error</td>
</tr>
<tr>
<td>Constant</td>
<td>10.27</td>
</tr>
<tr>
<td>Medium pressure extracting Steam rate per hour</td>
<td>0.522</td>
</tr>
<tr>
<td>Low pressure extracting Steam rate per hour</td>
<td>0.559</td>
</tr>
<tr>
<td>Generated power per hour</td>
<td>0.003</td>
</tr>
</tbody>
</table>

And Turbine #4 running model can be got as in (15)

$$S'_{4} = 0.522S_{4m} + 0.559S_{4L} + 0.003N_4 + 10.265$$  (15)

Where $190 \leq S'_{4} \leq 400$ t/h, $20 \leq S_{4m} \leq 65$, $30 \leq S_{4L} \leq 130$, $N_4 \leq 80000$. 

A. Turbine model

1) Turbine #4 model

Turbine #4 rated power is 80MW, principal steam temperature is 535°C and principal steam pressure is 12.3MPa. Based on the running data provided by the paper mill, the relationship among inlet steam amount, medium pressure extracting amount, low pressure extracting amount and generated power is as in Figure 1. From Figure 1, the linear relationship can be found among inlet steam amount and medium pressure extracting amount, low pressure extracting amount, low pressure extracting steam amount and generated power.

The regression model can be got as Table III with the help of SPSS (Statistical Package for the Social Sciences), and it has good linear relationship and the correlation coefficient is 0.998.
2) Turbine #1, 2, 3 models

The rated power of turbine #1, 2, 3 is 50MW, 80MW and 80MW, principal steam temperature is 535℃, principal steam pressure is 12.3MPa, and the regression models of turbine #1,2,3 are as in (16), (17) and (18)

Turbine#1:

\[ S_1 = 0.715S_{3m} + 0.443S_{1L} + 0.004N_1 + 13.954 \]  
(16)

Turbine #2:

\[ S_2 = 0.628S_{3m} + 0.395S_{2L} + 0.003N_2 + 18.211 \]  
(17)

Turbine #3:

\[ S_3 = 0.542S_{3m} + 0.449S_{3L} + 0.003N_3 + 17.427 \]  
(18)

where

\[ 140 \leq S_1 \leq 250, \ 5 \leq S_{1m} \leq 30, \ 35 \leq S_{1L} \leq 100, \ N_1 \leq 50000; \]

\[ 170 \leq S_2 \leq 330, \ 10 \leq S_{2m} \leq 45, \ 45 \leq S_{2L} \leq 140, \ N_2 \leq 80000; \]

\[ 130 \leq S_3 \leq 350, \ 10 \leq S_{3m} \leq 45, \ 45 \leq S_{3L} \leq 140, \ N_3 \leq 80000; \]

B. Boiler model

Boiler #1,2,3,4 are all circulating fluidized boiler and each boiler capacity parameters are as in Table II.

1) Boiler #4 running model

Boiler #4 rated evaporation capacity is 400t/h, principal steam temperature is 537℃, principal steam pressure is 12.5MPa, and the relationship between coal consumption and steam outlet is as in Figure 2.

From Figure 2 and Table IV, a good linear relationship between coal consumption and steam output for boiler #4 and \( R^2 \) is 0.96, \( F \) is 937.94.

Boiler #4 running model is as (19)

\[ B_4 = -0.768 + 0.17502S_4 - 0.0000475S_4^2 \]  
(19)

From Figure 2 and Table IV, a good linear relationship between coal consumption and steam outlet is as in Figure 2.

From Figure 2 and Table IV, a good linear relationship between coal consumption and steam output for boiler #4 and \( R^2 \) is 0.96, \( F \) is 937.94.

Boiler #4 running model is as (19)

\[ B_4 = -0.768 + 0.17502S_4 - 0.0000475S_4^2 \]  
(19)
\[ B_2 = -8.308 + 0.22482S_2 - 0.000125S_2^2 \]  
(21)

\[ B_3 = -10.134 + 0.24045S_3 - 0.000176S_3^2 \]  
(22)

Where, \(150 \leq S_1 \leq 240; \quad 230 \leq S_2 \leq 390; \quad 200 \leq S_3 \leq 400; \)

\(120 \leq S_4 \leq 390\)

### C. Optimization result

According to the power plant process structure of the paper mill, the supercontract can be obtained as in Figure 3. Based on the superstructure, the paper mill power plant non-linear programming model (NLPM) can be established. The objective constraint function is the minimum coal consumption, and medium pressure steam, low pressure steam and power demand refer to the real operation, and the other constraint can be got as energy balance calculation and material balance calculation.

![Fig.3 The superstructure of the paper mill power plant](https://doi.org/10.24084/repqj08.315)

Table V.-Optimization result of boilers

<table>
<thead>
<tr>
<th>Items</th>
<th>Boiler #1</th>
<th>After</th>
<th>Before</th>
<th>Boiler #2</th>
<th>After</th>
<th>Before</th>
<th>Boiler #3</th>
<th>After</th>
<th>Before</th>
<th>Boiler #4</th>
<th>After</th>
<th>Before</th>
<th>SUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal consumption, t/h</td>
<td>23.093</td>
<td>24.99</td>
<td>42.021</td>
<td>43.56</td>
<td>53.882</td>
<td>42.72</td>
<td>19.74</td>
<td>43.45</td>
<td>138.74</td>
<td>154.72</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steam output, t/h</td>
<td>150</td>
<td>177.06</td>
<td>262.04</td>
<td>266.34</td>
<td>362.67</td>
<td>264.21</td>
<td>121.16</td>
<td>263.73</td>
<td>895.87</td>
<td>971.34</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table VI.-Optimization result of turbines

<table>
<thead>
<tr>
<th>Items</th>
<th>Turbine #1</th>
<th>After</th>
<th>Before</th>
<th>Turbine #2</th>
<th>After</th>
<th>Before</th>
<th>Turbine #3</th>
<th>After</th>
<th>Before</th>
<th>Turbine #4</th>
<th>After</th>
<th>Before</th>
<th>SUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steam inlet, t/h</td>
<td>140</td>
<td>177.06</td>
<td>255.14</td>
<td>266.34</td>
<td>251.88</td>
<td>264.21</td>
<td>248.85</td>
<td>263.73</td>
<td>895.87</td>
<td>971.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium pressure extracting steam rate, t/h</td>
<td>5</td>
<td>15.35</td>
<td>10</td>
<td>27.97</td>
<td>29.79</td>
<td>33.04</td>
<td>65</td>
<td>33.43</td>
<td>109.79</td>
<td>109.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low pressure extracting steam rate, t/h</td>
<td>85</td>
<td>69.57</td>
<td>140</td>
<td>73.29</td>
<td>100.65</td>
<td>107.20</td>
<td>30</td>
<td>104.40</td>
<td>354.46</td>
<td>354.46</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generated power, kwh</td>
<td>21204</td>
<td>53690</td>
<td>58451</td>
<td>45597</td>
<td>57883</td>
<td>57323</td>
<td>62628</td>
<td>61556</td>
<td>200166</td>
<td>200166</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The established model is a NLPM, and there are twenty seven variables, fifteen equation constraint, four inequation constraint and eight independent variables. Aid with MATLAB non-linear programming function, fmincon, the optimization result can be got as Table V, and Table VI.

From Table V, the coal consumption after optimization is 138.74 t/h and it is 154.72 t/h before optimization, that’s to say, 15.98 t/h coal is saved ,and the steam output is 895.87 t/h. And from Table VI, when feeding 895.87 t/h steam, the turbines also can produce the same medium pressure extraction steam, low pressure extracting steam and power.

According to the optimization result, when adjusting the boiler and turbines load distribution, the minimum coal consumption can be obtained.

5. Conclusion

The boiler and turbine load distribution optimization is an important method for paper mill power plant economical operation and energy saving, it can reduce the energy consumption and exhaust emission and improve profile and environment.

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References


