A NEW DESIGN METHODOLOGY OF A SWITCHED RELUCTANCE GENERATOR CONSIDERING THE INFLUENCE OF THE MUTUAL INDUCTANCES


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Abstract – The subject of this paper is to present the design of a Switched Reluctance Generator (SRG) taking into account the presence of mutual inductance, which has been ignored in the literature of switched reluctance machine (SRM). A step-by-step for the mechanical and electrical magnitudes sizing is demonstrated, both for prototype construction and for graphic representation in CAD software. Thus, with the drawing of SRM lamination plate, the mutual inductance is measured through a finite element software called FEMM (Finite Element Method Magnetics), whose values are processed and arranged in inductance matrices with the aid of MATLAB®. Once with inductance matrices, the mathematical modeling is implemented on SIMULINK MATLAB® and the results are presented. Finally a SRG prototype is constructed to validate the design proposed here.

Keywords – Electrical Machines Design, Switched Reluctance Generator, Mutual Inductance, FEMM, MATLAB®.

I. INTRODUCTION

The Switched Reluctance Machine (SRM) has been known since the nineteenth century, but the difficulty in obtaining effective control eventually get it from the market. Only with recent advances in power electronics, the sensing and microprocessor SRM has become feasible, reliable and efficient. It is a robust machine, simpler construction compared to conventional electric machines. Its biggest advantage is associated with the performance variable speed system, with an option to use alternative energy sources such as wind. [1,3]

Thus the crescent need of precision to control the SRM and to apply it on more accurate systems, it is consistent to consider the mutual inductance because of its intrinsic nature on electrical machines.

A 6x4 Switched Reluctance Machine owned a laminated double salient-pole structure with limited winding to the stator poles. Each coil of a pair of opposite poles consists a phase, so that the SRM has three-phase configuration. [3]. The figure Fig.1 follows with the representation of the rotor and stator lamination plates and the winding of phase A.

Fig. 1. Lamination plate of a SRM 6x4 and phase winding

The drive of each of the three phases, consisting of a series circuit coursed by a pulsed direct current is made by the shooting of half-bridge inverter switches. The shooting time must take into account the rotor position in order to determine the operation of the SRM as motor or generator [1,3]. In the present work we use an industrial encoder for determining the angular position. Importantly, the phase alignment position A is adopted as reference (zero degree position) in this work.

To drive the SRM as motor the trigger of each converter switch, which is controlled by the positioning sensor must be closed when the respective phase is completely misaligned, that is, 45 degrees of the reference position. The actuation of the phase A coil will last for 30 degrees, that way, the missing 15 degrees to full alignment. This action is taken so that the winding current is extinguished until the zero degree position and does not cause a negative torque. In this position (15 degrees of alignment) the energy stored in the coil of phase A will be returned to the source. The drive of the other phases proceeds in the same manner.

To trigger SRM in generator mode will also be used a half-bridge converter, but with the presence of a load at its terminals. For Switched Reluctance Generator the free wheel
happens to be among the coils of each phase and the load, so that it is constituted as the power generation stage itself.

In half-bridge Generator the trigger of each key also controlled by the positioning sensor is driven so that the phase is energized 4.7° before complete alignment. During 30 degrees occurs energizing phase, being the same turned off in 25.3°. The energy stored in phase A is released to the load. The other two phases are powered in the same manner. The focus of this work is to trigger the machine Reluctance in the generator mode.

II. MATHEMATICAL MODEL

From the understanding of the principles of operation of the Switched Reluctance Generator, it is possible a mathematical formulation of the running of these converters.

A. Electric Equations

In the first instance, one must consider the relationship between the looped magnetic flux (\( \lambda \)) by the circuit coils and the instantaneous current (\( i \)) applied to them, and therefore regarded as self inductance (\( L \)) of the circuit.

\[
L = \frac{\lambda}{i} \tag{1}
\]

A variation of this flux linkage versus time is proportional to the induced voltage (\( e \)):

\[
e = \frac{d\lambda}{dt} \tag{2}
\]

Thus, the voltage at the terminals of each phase has resistive nature by the presence of conductors; and inductive nature due to changes in flux. Thus, for a phase j:

\[
v_j = R_i j + L \frac{di_j}{dt} \tag{3}
\]

Since, however, the flux linkages is proportional to the inductance and the current and the solution of equation (3) involves a partial derivative where \( L \) is first considered constant and \( i \) variable, and then the opposite. And as \( L \) varies in relation to the angular position and time, equation (3) results in :

\[
v_j = R_i j + L \frac{di_j}{dt} + i_j \omega \frac{dL_j}{d\theta} \tag{4}
\]

Equation (4) describes a phase electrically SRM; however, for the three phases it is necessary to consider that the magnetic flux comes from the current stage itself and current coming from the other phases. It has then:

\[
\lambda_1 = L_{11} i_1 + L_{12} i_2 + L_{13} i_3 \tag{5}
\]

\[
\lambda_2 = L_{21} i_1 + L_{22} i_2 + L_{23} i_3
\]

\[
\lambda_3 = L_{31} i_1 + L_{32} i_2 + L_{33} i_3
\]

It is noteworthy that in this study the mutual inductances are considered. Now, substituting in equation (3) the group (5) equations, we have:

\[
v_1 = R_1 i_1 + \frac{dL_{11} i_1}{dt} + \frac{dL_{12} i_2}{dt} + \frac{dL_{13} i_3}{dt}
\]

\[
v_2 = R_2 i_2 + \frac{dL_{21} i_1}{dt} + \frac{dL_{22} i_2}{dt} + \frac{dL_{23} i_3}{dt}
\]

\[
v_3 = R_3 i_3 + \frac{dL_{31} i_1}{dt} + \frac{dL_{32} i_2}{dt} + \frac{dL_{33} i_3}{dt}
\]

Solving the derivatives of equation (6) results and arranging it in matrix form, we obtain the equation (7). This equation describes the SRM only electrically. This creates thus a need in balancing the mechanical forces which will be described in the next section.

\[
\begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix} = 
\begin{bmatrix}
R_1 + \omega \frac{\partial L_{11}}{\partial \theta} & \omega \frac{\partial L_{12}}{\partial \theta} & \omega \frac{\partial L_{13}}{\partial \theta} \\
\omega \frac{\partial L_{21}}{\partial \theta} & R_2 + \omega \frac{\partial L_{22}}{\partial \theta} & \omega \frac{\partial L_{23}}{\partial \theta} \\
\omega \frac{\partial L_{31}}{\partial \theta} & \omega \frac{\partial L_{32}}{\partial \theta} & R_3 + \frac{1}{2} \omega \frac{\partial L_{33}}{\partial \theta}
\end{bmatrix} 
\times
\begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix}
\tag{7}
\]

B. Mechanical addressing

When the involvement of mechanical forces, the equation depends on the balance between them. Therefore, in the case of the generator, the mechanical energy - a mechanical conjugated form (\( T_{mech} \)) - entering the system shall be able to overcome electromagnetic torque (\( T_{emag} \)), the rotational inertia (\( J \frac{d\omega}{dt} \)) and the dynamic friction bearing (\( D \frac{d\omega}{dt} \)), as follows:

\[
T_m = T_{emag} + D \omega + J \frac{d\omega}{dt}
\tag{8}
\]

To calculate the electromagnetic torque (\( T_{emag} \)), it is handed the concept of co-energy. As the instantaneous electromagnetic torque is given by:

\[
T_{emag} = \frac{dW_{emag}(i, \theta)}{d\theta}
\tag{9}
\]

That for the three phases of SRM results in:

\[
T_{emag} = \frac{dW_{1emag}}{d\theta} + \frac{dW_{2emag}}{d\theta} + \frac{dW_{3emag}}{d\theta}
\tag{10}
\]

Taking into consideration the equality \( W = \frac{1}{2L} \lambda^2 \), it had to:

\[
W_1 = \frac{1}{2} (L_{11} i_1^2 + L_{12} i_2^2 + L_{13} i_3^2)
\tag{11}
\]

\[
W_2 = \frac{1}{2} (L_{21} i_1^2 + L_{22} i_2^2 + L_{23} i_3^2)
\tag{11}
\]

\[
W_3 = \frac{1}{2} (L_{31} i_1^2 + L_{32} i_2^2 + L_{33} i_3^2)
\tag{11}
\]

The total co-energy of the system is the sum of the equations (13). Substituting this sum in equation (11) and
solving the derivatives and substituting into equation (8) we have:

\[ T_{\text{emag}} = \frac{1}{2} \left[ i_1 \left( \frac{\partial L_{11}}{\partial \theta} \frac{\partial L_{12}}{\partial \theta} i_1 + \frac{\partial L_{12}}{\partial \theta} i_1 + \frac{\partial L_{13}}{\partial \theta} i_1 \right) + \right. \\
\left. + i_2 \left( \frac{\partial L_{12}}{\partial \theta} i_2 + \frac{\partial L_{22}}{\partial \theta} i_2 + \frac{\partial L_{23}}{\partial \theta} i_2 \right) + i_3 \left( \frac{\partial L_{13}}{\partial \theta} i_3 + \frac{\partial L_{23}}{\partial \theta} i_3 + \frac{\partial L_{33}}{\partial \theta} i_3 \right) + D \omega f \frac{d \omega}{dt} \right] \]  

(12)

C. States Matrix

The array of states is, in short, the mechanical and electromagnetic behavior of the three-phase SRM, is for the generator:

\[
\begin{pmatrix}
v_1 \\
v_2 \\
v_3 \\
T_m \\
0
\end{pmatrix} = \begin{bmatrix}
R_1 & \omega \frac{\partial L_{12}}{\partial \theta} & \omega \frac{\partial L_{13}}{\partial \theta} & 0 & 0 \\
\omega \frac{\partial L_{21}}{\partial \theta} & R_2 & \omega \frac{\partial L_{23}}{\partial \theta} & 0 & 0 \\
\omega \frac{\partial L_{31}}{\partial \theta} & \omega \frac{\partial L_{32}}{\partial \theta} & R_3 & 0 & 0 \\
\frac{1}{2} i_1 X_1 & \frac{1}{2} i_2 X_2 & \frac{1}{2} i_3 X_3 & 0 & D \\
0 & 0 & 0 & -1 & 0
\end{bmatrix} \begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
\omega
\end{bmatrix}
\]

(13)

where: \( X1 = \frac{\partial L_{11}}{\partial \theta} + \frac{\partial L_{12}}{\partial \theta} + \frac{\partial L_{13}}{\partial \theta} \)

\( X2 = \frac{\partial L_{12}}{\partial \theta} + \frac{\partial L_{22}}{\partial \theta} + \frac{\partial L_{23}}{\partial \theta} \)

\( X3 = \frac{\partial L_{13}}{\partial \theta} + \frac{\partial L_{23}}{\partial \theta} + \frac{\partial L_{33}}{\partial \theta} \)

III. DESIGN OF DIMENSIONS

Equipped with the equations describing the operation of the machine, the next step is to the setting of physical and electrical parameters of SRM.

D. Definition of power

The first step in design of electric machines is knowledge of the mechanical characteristics of the load, that will be triggered. In this case, it is desired to design a machine that can drive a load of 1 hp with a rated speed of 1200 rpm.

Being one c.v. \( \cong 735.5 \) W, 1 rpm = \( 2 \pi / 60 \) rad / s and power is the energy or labor expense per unit of time, then:

\[ P = \frac{\Delta \theta}{\Delta t} (W) \text{ and } T = F. \Delta \theta \text{ (J)} \]  

(14)(15)

Where \( \Delta \theta \) is the angular displacement. Substituting:

\[ P = \frac{F. \Delta \theta}{\Delta t} \text{ (W)} \]  

(16)

Conceptually, \( \frac{\Delta \theta}{\Delta t} \) is the angular velocity (\( \omega \) in rad/s, and the forces involved in rotating machines are rotary forces. So force (F) in the above equations must be understood as conjugate (C), so that:

\[ P = C. \omega \text{ (W)} \]  

(17)

As \( P = 735.5 \) W and \( \omega = 125.664 \) rad/s, then: you want to design a machine that provides a conjugate in its axis of \( C = 5.8529 \) Nm.

The equation relating to electromagnetic origin quantities (F and B) with geometrical machine (\( D_r \) = rotor diameter) and (\( \text{Ir} = \text{rotor length} \)) is:

\[ F = A_e * B_e * \pi * D_r * l_r \text{ (N)} \]  

(18)

Once it found the force acting on the rotor surface, just multiply by the rotor radius to find the conjugate. Thus, it follows that the torque (C) supplied by the machine will be:

\[ C = \frac{1}{2} A_e * B_e * \pi * D_r^2 * l_r \text{ (Nm)} \]  

(19)

In general, equation (22) is written on a constant which is proportional to the electrical load (\( A_e \)) and the electromagnetic load (\( B_e \)), and relate to the conjugate per rotor volume (MVT) given in Nm/m. Therefore, the equation (22) takes the following form:

\[ C = k * D_r^2 * l_r \text{ (Nm)} \]  

(20)

Where \( k = \frac{\text{VMT} \cdot \pi}{4} \).

Having the equation (20), which provides the conjugate of the machine, to obtain power, the phrase is multiplied by the angular (\( \omega \)) specified:

\[ P = \frac{\text{VMT} \cdot \pi}{4} * D_r^2 * l_r * \omega \text{ (W)} \]  

(21)

The ratio between the parameters \( D_r \) and \( l_r \) is defined as the variable, which denotes the machine format showing the relationship between diameter and length. Making the appropriate substitutions and isolating the variable (Dr), one has:

\[ D_r = 3 \sqrt{\frac{4 \cdot P}{\text{VMT} \cdot \pi \cdot K_1 \cdot \omega}} = 3 \sqrt{\frac{4 \cdot 735.5}{16000 \cdot \pi \cdot 1.6355 \cdot 125.664}} = 0.07 \text{ (m)} \]  

(22)

This work will use one k1 of 1,6355, that is, the length of the machine will be 60 % greater than the diameter. And VMT is taken from [4] and the value is used for industrial applications 16000 engines. Thus, it is has all the data necessary for determining the rotor of the machine design.

Therefore, the length of the machine will be:

\[ l_r = k_1 \cdot D_r \cong 0.1147 \text{ m} \]  

E. Choose the stator pole angle

According to [4], the maximum angle of the stator that can produce the phase conjugate nonzero is given by 180°
divided by the number of poles (Nps) it is desirable that the stator has. In this case, the stator of the angle will be:
\[ \beta_s = \frac{180}{6} = 30 \text{ (degrees)} \] (23)

**F. Gap length**

According to [4] the value of the gap (g) should be around 0.5 % of the rotor diameter. To increase the flow, the gap was considered as small as possible, in this work the gap is 0.24 % of the impeller diameter; therefore:
\[ g = 0.0024 \times D_r \equiv 0.0003 \text{ (m)} \] (24)

**G. Stator yoke height**

This height must be sufficient so that the magnetic flux density in the yoke is not greater than the magnetic flux density in the stator poles. By knowledge of the physical structure of the machine, set forth in Figure 1, it is estimated that the magnetic flux passing through the stator poles divided in the breech approximately half going to each side. Thus, the height of the yoke (ys) must be at least half the stator pole width.

Another observation to be made from this time Figure 4: the stator pole width is equal to the sine of the angle \( \beta_s \). How you want to discover half of this value, you must use half -angle of 30 ° \( (\beta_s) \).

\[ \beta_s = \frac{\theta_s}{2} \]

**H. External stator diameter**

From the choice of substrate 90S which has an inner diameter of 0.1401 m external diameter of the stator was set to this value, so that the slides to be accommodated.

**I. Stator pole width**

Making use again in Figure 4, it was found that the value of the stator yoke should be half the stator pole width (ts), then:
\[ t_s \equiv 2 \times 0.0122 \equiv 0.02 \text{ m} \]

**J. Rotor pole width**

The rotor pole width must be larger than the stator pole width to allow for an increase in residence in the position of full alignment of the stator poles and rotor and decrease the spreading. It is common to see an increase of 2* the rotor pole, then:
\[ t_r = t_s + 2 \times g = 0.02 + 2 \times 0.0003 = 0.0206 \text{ (m)} \] (26)

**K. Rotor pole angle**

Since it is known rotor pole width (tr) and the rotor diameter (Dr), then the rotor pole angle \( (\beta_r) \) is approximately the angle whose sine is given by the width divided by the radius:
\[ \beta_r = 2 \times \arcsin \left( \frac{L}{D_r} \right) \equiv 34.1791° \] (27)

**L. Rotor yoke height**

As the torque produced by a VRM is proportional to the inductance variation of the angular position relative to obtain this effect, the height of the rotor pole \( (dr) \) should be at least 20 to 30 times the length of the gap (g) [4].

Also according to [4], in practice, the rotor pole may be half the height of the stator pole width (ts), so:
\[ d_r = \frac{t_r}{2} \equiv 0.012 \text{ m} \] (28)

**M. Rotor yoke height**

The rotor yoke (yr) should be at least half the rotor tooth width (tr). Reference [4] recommend an increase of 20 % to 40% of this value. In this paper we used a 20% increase.
\[ y_r = \frac{t_r}{2} \equiv 0.0124 \text{ m} \] (29)

**N. Rotor shaft diameter**

The shaft diameter can be defined by the following equation:
\[ D_{ext} = D_r - 2 \times (d_r + y_r) \equiv 0.0210 \text{ m} \] (30)

**O. Stator pole height**

The stator pole height can be logically determined using the data obtained so far, so that:
\[ d_s = \frac{t_s}{2} (D_s - D_r - 2 \times (g + y_s)) = 0.0225 \text{ m} \] (31)

**P. Calculation of the number of turns of each phase**

Set the machine power in item (F) of approximately 735,5W and considering a yield of 70 %, the input power is:
\[ P_e \times \eta = P_s = 1.0507 \text{ kW} \] (32)

In this work, it was considered that the rectifier used to power the converter provides a voltage approximate 180V dc, which is the input voltage \( (V_e) \) system. The input current \( (i_e) \) of the system is:
\[ P_e = V_e \times i_e \equiv 5.8 \text{ (W)} \] (33)

This chain is responsible for the appearance of the magnetic field in the coils SRM, whose magnetomotive force \( (F_{mn}) \) is proportional to the current passing through the coils and the number of turns \( (N_r) \) in that coil.
\[ F_{mn} = i_e \times N_e \text{ (ampère-espiras)} \] (34)

From the Circuits Ampere's Law, which states that the line integral of the magnetic field intensity \( (H) \) along your route
results in the magnetomotive force, performs the substitution in equation (35):

\[ i_e \times N_e = \oint H \, \text{d}l \]  

(35)

For calculation purposes, the magnetic flux density was kept constant throughout the magnetic circuit, so that a adopted \( B_{\text{ent}} = 1.2 \ \text{T} \ (\text{Hent}) \). The magnetic field strength in the air gap (\( H_{\text{ent}} \)) can be found by equation (36):

\[ H_{\text{ent}} = \frac{B_{\text{ent}}}{\mu_0} \ (\text{A.e/m}) \]  

(36)

Where \( \mu_0 = 4 \times \pi \times 10^{-7} \ \text{Tm/A.e} \):

\[ H_{\text{ent}} \approx \frac{1.2}{4 \times \pi \times 10^{-7}} \approx 9.55 \times 10^5 \ \text{A.e/m} \]

The reluctance of the ferromagnetic material was considered negligible compared to the reluctance of the air gap, disregarding their contribution in value of \( F_{\text{mm}} \cdot \text{dL} \) is considered constant and has the value of two air gaps (2\( g \)), which is 0.0006 meters.

With the necessary replacement, equation (37) takes the following:

\[ i_e \times N_e = H_{\text{ent}} \times 2 \times g = 98 \ \text{turns} \]  

(37)

Therefore, one should choose a wire that supports the calculated current. For this project, it used the 10 AWG.

IV. OBTAINING SELF AND MUTUAL INDUCTANCE USING SOFTWARE FEMM

Self and mutual inductance profiles, as well as their derivatives, are input to the simulation performed in MATLAB®/SIMULINK.

The designs of the lamination (Figure 4) were imported from AutoCAD software in DXF format and settings problem - as materials, circuits and boundary conditions - were defined in FEMM.

After defining these parameters, a command sequence is created in Lua script, a programming language designed to meet the processing procedures available in FEMM. The algorithm is designed so that it is established current value from 1 to 45 amps and for each of them is performed in the rotor angular displacement of 0 to 90°. For each pair of values (current - angle) self and mutual inductances are calculated, resulting the arrays needed for the simulation. Figures 3, 4 and 5 refer to the plots of inductances MATLAB®.

Fig. 3. Surface own inductance of phase A SRM 6x4

Fig. 4. Surface mutual inductance of the coil phase the phase B of SRM 6x4

Fig. 5. Surface of mutual inductances of the phase A coil at phase C

V. SIMULATION ON MATLAB / SIMULINK AND COMPARISON WITH EXPERIMENTAL RESULTS

The computational solution of mathematical model of the SRG is developed on the variable time through functional SIMULINK blocks and programs implemented with the appropriate design values. The SIMULINK blocks, simulation graphical environment, represent each stage of operation of the generator and its physical components. These blocks have the operation governed by the .m files, which are the algorithms implemented in MATLAB.

The built test bench (Figure 6), will consist of a machine to Reluctance (driven by a half-bridge three-arm converter), directly connected to the shaft of a three-phase induction motor 3 hp, which will be used as a mechanical force input to the generator. This machine was thrown likewise done in the simulations. The aim is to compare the results of the prototype computer simulations and validate the design of a Generator Reluctance.
The generating mode is simulated and the current in one of the phases is measured and shown in Figure 7. Figure 8 shows the calibration counter.

In Figure 8 it used a Hall effect current sensor calibrated to 1 V at 2.5 A output when the input. Therefore, the amplitude of the current in the phase SRG is approximately 15 A.

As can be seen Figures 7 and 8 show coherence both in amplitude and waveform.

The same comparison is made on the voltage on one of the phases respectively the simulation result in Figure 9 and the workbench in Figure 10. It can be seen that Figures 9 and 10 show the same amplitude values and the same behavior.

VI. CONCLUSIONS

The design methodology presented in this work was to differential consideration of the influence of mutual inductances in the development of the Switched Reluctance Generator (SRG). This project had the intention of providing a methodology that reaches a design as similar as possible to the prototype.

All found data in the built prototype proved consistent with the results taken from simulations. Both current values and experimentally found the tension had been predicted in the simulation. The SRG built generated exactly 1 c.v. when it was at 1200 rpm. Soon, the project presented in this work proved to be extremely accurate and can be used to build other Switched Reluctance Machines. Moreover, the consistency of the methodology can lead to improvements on SRM design in the future.

REFERENCES


