

Cost Loss Allocation in Distribution Networks with High Penetration of Distributed Renewable Generation – A Comparative Study

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Abstract. The implementation of market mechanisms to remunerate distributed generation should take into account a non-discriminatory access to distribution networks. In consequence, power losses of distribution network must be fairly allocated among the all distributed generators and consumers. Several methods for power loss cost allocation have been proposed in the literature, divided basically into two groups. Firstly, methods as postage stamp, mw-mile, circuit based and proportional sharing have been supported on an arbitrary allocation of power losses between consumers and generators, typically 50-50%. More recently, a modified proportional sharing procedure has been proposed based on the allocation of the entire losses to consumers disregarding the influence of distributed generators using the basic proportional sharing principle and reallocate avoided or produced losses among distributed generators. Secondly, marginal procedures have been extensively proposed in order to send efficient economical signals to the market agents. Marginal methods require a slack bus designation and do not assign arbitrarily power losses among producers and consumers. This paper presents a comparative study of different loss allocation procedures taking into account different levels of penetration of renewable sources in distribution networks. Results are obtained and discussed from a test distribution network.

Key words

Loss allocation, distribution systems, distributed generation, dispersed generation, embedded generation.

1. Introduction

Under ongoing restructuring process, electric distribution activities remains as monopoly and traditional regulatory schemes based in Cost of Service (CoS) have been substituted by new regulatory frameworks as Performance Based Ratemaking (PBR) which are developed to encourage economic efficiency, proposing an open and non-discriminatory access to the networks. Unfortunately, an open access policy must be made through an imperfect delivery network that produces power losses. In consequence, these power losses must be fairly allocated among the all distributed generators and consumers. Losses are nonlinear functions of line flows and it is difficult determining the responsibility of each generator and demand in global power losses.

These facts permit the existence of several and different methods for power loss cost allocation in the literature, mainly in transmission systems [1]. A lack of information is observed about allocation methods applied to distribution systems. Generally, power losses at distribution level were considered as an additional load and therefore allocated among all consumers using average values [10]. However, a huge increase of distributed generation forces the development of new allocation procedures and some ones have been proposed recently, usually adapted from methodologies and widely applied in transmission systems. Basically, taking into account the way that power losses are allocated among network users these methods could be divided into two groups.

Firstly, procedures as postage stamp [2],[3], MW-mile [2],[3], Zbus [4] and proportional sharing [5] are based on an arbitrary distribution of power losses between consumers and generators, typically 50-50%. More recently, a modified proportional sharing procedure has been proposed [6] based on the allocation of the entire losses to consumers disregarding the generators influence using the proportional sharing principle and reallocate avoided or produced losses among distributed generators. Secondly, marginal procedures were extensively used in transmission systems [7]. Well known Incremental Transmission Coefficients (ITL) have been modified and introduced as Marginal Loss Coefficients (MLCs) in order to allocate power losses in distribution systems with distributed generation [8],[9].

The growing penetration of distributed generation into distribution networks essentially though renewable generation must be properly assessed in order to verify its impact in the allocation of global avoided or produced losses in the system among all market agents. This paper presents a comparative study of seven loss allocation procedures taking into account different levels of penetration of distributed generation in distribution networks. The methods studied were 1) Marginal, 2) Reconciled Marginal, 3) Zbus, 4) Postage stamp, 5) MW-mile, 6) Proportional Sharing, 7) Proportional Sharing Modified. Results were obtained and discussed from a test distribution network with two distributed generators.

2. Cost loss allocation methodologies

Loss allocation procedure consists in assign total active power loss L among all generators and demands:

$$L = \sum_{i=1}^n L_{Gi} + \sum_{i=1}^n L_{Di} \quad (1)$$

where

L_{Gi} active power loss allocated to generators of bus i ;

L_{Di} active power loss allocated to demands of bus i .

n number of buses.

A. Postage Stamp

The postage stamp procedure use average values to proportionally allocate 50% of losses to the demands and 50% to generators, that is [3]:

$$L_{Di} = 50\% \times L \times P_{Di} / P_D, \quad i = 1, \dots, n \quad (2)$$

$$L_{Gi} = 50\% \times L \times P_{Gi} / P_G, \quad i = 1, \dots, n \quad (3)$$

where

P_{Gi} active power output of generators of bus i ;

P_{Di} active power input of demands of bus i ;

P_G, P_D total active power generated and demanded;

B. Power Flow based MW-mile

The power flow based MW-mile procedure allocates 50% of losses to the demands and 50% to generators and are based on the extent of use of distribution network by all generators and demands. Then, allocation value by use of the network T applied to each demand and generator is expressed as [2]:

$$T_{Di} = \sum_{k=1}^m F_{Dik} l_k \quad (4)$$

$$T_{Gi} = \sum_{k=1}^m F_{Gki} l_k \quad (5)$$

where,

F_{Dkj} power flow in branch k produced by demand i

F_{Gkj} power flow in branch k produced by generator i

l_k length of branch k

m number of branches

Losses allocated to demands and generators are given by:

$$L_{Di} = 50\% LT_{Di} / \sum_{j=1}^m T_{Dj} \quad i = 1, \dots, n \quad (6)$$

$$L_{Gi} = 50\% LT_{Gi} / \sum_{j=1}^m T_{Gj} \quad i = 1, \dots, n \quad (7)$$

C. Proportional Sharing

Proportional sharing methods [5] are based on gross line flows. This method assumes that the system is fed with the actual generation, and then nodal generation remains unchanged but nodal demands must be slightly modified in order to accomplish Kirchoff's laws. However, it is not possible, to allocate losses to generators and demands at the same time. Generally, upstream looking algorithm is applied to allocate 50% power losses between generators and a downstream looking algorithm to allocate the other 50% of power losses between demands.

1) Downstream looking algorithm

The total power gross flow P_i^{gross} (i.e. the sum of all outflows at node i) may be expressed as:

$$P_i^{gross} = P_{Gi} + \sum_{j \in \alpha_i^{down}} \delta_{ji} P_j^{gross} \quad i=1, \dots, n \quad (8)$$

$$\delta_{ji} = P_{ji}^{gross} / P_j^{gross} \quad (9)$$

where,

$\sum_{j \in \alpha_i^{down}} \delta_{ji} P_j^{gross}$ power flow leaving bus i ;

α_i^{down} set of buses receiving power from bus i ;

Equation (8) constitutes a system of linear equations and may be written as:

$$A_d P^{gross} = P_G \quad (10)$$

Then, gross demand at bus i , can be expressed:

$$P_{Di}^{gross} = \frac{P_{Gi}}{P_i} \sum_{j=1}^n [A_{dij}^{-1}] P_{Dj} \quad i=1, \dots, n \quad (11)$$

where P_i is the real injection i given by a power flow solution and power losses allocated to generators are:

$$L_{Gi} = P_{Gi}^{gross} - P_{Gi} \quad i=1, \dots, n \quad (12)$$

2) Upstream looking algorithm

The total power gross flow P_i^{gross} (i.e. the sum of all inflows at node i) may be expressed as:

$$P_i^{gross} = P_{Gi} + \sum_{j \in \alpha_i^{up}} \delta_{ji} P_j^{gross} \quad i=1, \dots, n \quad (13)$$

$$\delta_{ji} = P_{ji}^{gross} / P_j^{gross} \quad (14)$$

where,

$\sum_{j \in \alpha_i^{up}} \delta_{ji} P_j^{gross}$ power flow reaching bus i from lines connected it;

α_i^{up} set of buses from which power flows toward bus i ;

Equation (13) constitutes a system of linear equations written as:

$$A_u P^{gross} = P_G \quad (15)$$

Then, gross demand at bus i , can be expressed:

$$P_{Di}^{gross} = \frac{P_{Di}}{P_i} \sum_{j=1}^n [A_{uij}^{-1}] P_{Gj} \quad i=1, \dots, n \quad (16)$$

where P_i is the real injection i given by a power flow solution and power losses allocated to demand are:

$$L_{Di} = P_{Di}^{gross} - P_{Di} \quad i=1, \dots, n \quad (17)$$

D. Proportional Sharing Modified

This method [6] is based on proportional sharing principle presented in [5] but with a different assignation of losses. This approach consists in to allocate the entire losses (100%) to demands disregarding the influence distributed generators using the proportional sharing principle and reallocate avoided or produced losses among distributed generators. The implementation of this method requires a load flow solution with and without distributed generators in order to compute avoided or produced loss of overall distributed injections.

E. Circuit Based Allocation (Zbus)

Circuit based loss allocation model [4] distribute the system losses L among n buses of the network according to:

$$L = \sum_{i=1}^n L_i \quad (18)$$

L_i components are obtained from the real part of the inverse of admittance matrix $Y=G+jB$, $Z_{bus} = Y^{-1} = R+jX$ through the following general expression:

$$L_i = \Re \left[I_i^* \left(\sum_{j=1}^n R_{ij} I_j \right) \right] \quad i=1, \dots, n \quad (19)$$

where, $I_i = (S_i^*/V_i^*)$.

In general, if a given bus i has both generation P_{Gi} and demand P_{Di} , the allocation loss component L_i can be further assigned in a *pro rata* manner.

$$L_{Gi} = (1 - \gamma_i) L_i, \quad L_{Di} = \gamma_i L_i \quad i=1, \dots, n \quad (20)$$

where,

$$\gamma_i = P_{Di} / (P_{Gi} + P_{Di}) \quad i=1, \dots, n \quad (21)$$

F. Marginal Allocation

Under incremental analysis power losses can be allocated to producers and consumers simultaneously through Marginal Loss Coefficients (MLCs) at given operating point. By definition, these coefficients measure the change in power active losses due to incremental change in power injections in each node if the network [1]:

$$MLC_i^p = \frac{\partial L}{\partial (P_{Gi} - P_{Di})} \quad MLC_i^o = \frac{\partial L}{\partial (Q_{Gi} - Q_{Di})} \quad (22)$$

As there is no explicit relationship between losses and power injections the standard chain rule is applied in the calculation of MLCs using intermediate variables, voltages and angles obtained from a converged AC power flow [7]:

$$\begin{bmatrix} MLC_{AC}^p \\ MLC_{AC}^o \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \theta_1} & \frac{\partial P_2}{\partial \theta_1} & \dots & \frac{\partial P_n}{\partial \theta_1} & \frac{\partial Q_1}{\partial \theta_1} & \frac{\partial Q_2}{\partial \theta_1} & \dots & \frac{\partial Q_n}{\partial \theta_1} & \frac{\partial L}{\partial \theta_1} \\ \frac{\partial P_1}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_2} & \dots & \frac{\partial P_n}{\partial \theta_2} & \frac{\partial Q_1}{\partial \theta_2} & \frac{\partial Q_2}{\partial \theta_2} & \dots & \frac{\partial Q_n}{\partial \theta_2} & \frac{\partial L}{\partial \theta_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial P_1}{\partial \theta_n} & \frac{\partial P_2}{\partial \theta_n} & \dots & \frac{\partial P_n}{\partial \theta_n} & \frac{\partial Q_1}{\partial \theta_n} & \frac{\partial Q_2}{\partial \theta_n} & \dots & \frac{\partial Q_n}{\partial \theta_n} & \frac{\partial L}{\partial \theta_n} \\ \frac{\partial P_1}{\partial V_1} & \frac{\partial P_2}{\partial V_1} & \dots & \frac{\partial P_n}{\partial V_1} & \frac{\partial Q_1}{\partial V_1} & \frac{\partial Q_2}{\partial V_1} & \dots & \frac{\partial Q_n}{\partial V_1} & \frac{\partial L}{\partial V_1} \\ \frac{\partial P_1}{\partial V_2} & \frac{\partial P_2}{\partial V_2} & \dots & \frac{\partial P_n}{\partial V_2} & \frac{\partial Q_1}{\partial V_2} & \frac{\partial Q_2}{\partial V_2} & \dots & \frac{\partial Q_n}{\partial V_2} & \frac{\partial L}{\partial V_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial P_1}{\partial V_n} & \frac{\partial P_2}{\partial V_n} & \dots & \frac{\partial P_n}{\partial V_n} & \frac{\partial Q_1}{\partial V_n} & \frac{\partial Q_2}{\partial V_n} & \dots & \frac{\partial Q_n}{\partial V_n} & \frac{\partial L}{\partial V_n} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \theta_1} \\ \frac{\partial L}{\partial \theta_2} \\ \vdots \\ \frac{\partial L}{\partial \theta_n} \\ \frac{\partial L}{\partial V_1} \\ \frac{\partial L}{\partial V_2} \\ \vdots \\ \frac{\partial L}{\partial V_n} \end{bmatrix} \quad (23)$$

The general system of linear equations is solved eliminating the columns and rows correspondent to slack and PV nodes: $MLC_{sl}^p = MLC_{sl}^o = MLC_{pv}^o = 0$ and entries of the transpose of Jacobian and the right-hand vector are:

$$\frac{\partial P_i}{\partial \theta_j} = V_i V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (24)$$

$$\frac{\partial P_i}{\partial \theta_i} = -B_{ii} V_i^2 - \sum_{j=1}^n V_i V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (25)$$

$$\frac{\partial P_i}{\partial V_j} = V_i [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (26)$$

$$\frac{\partial P_i}{\partial V_i} = G_{ii} V_i + \sum_{j=1}^n V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (27)$$

$$\frac{\partial Q_i}{\partial \theta_j} = -V_i V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (28)$$

$$\frac{\partial Q_i}{\partial \theta_i} = -G_{ii} V_i^2 + \sum_{j=1}^n V_i V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (29)$$

$$\frac{\partial Q_i}{\partial V_j} = V_i [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (30)$$

$$\frac{\partial Q_i}{\partial V_i} = -B_{ii} V_i + \sum_{j=1}^n V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (31)$$

$$\frac{\partial L}{\partial \theta_i} = 2 \sum_{j=1}^n V_i V_j G_{ij} \sin(\theta_i - \theta_j) \quad (32)$$

$$\frac{\partial L}{\partial V_i} = 2 \sum_{j=1}^n G_{ij} [V_i - V_j \cos(\theta_i - \theta_j)] \quad (33)$$

First computations of losses allocated to generators and demands connected to bus i are, respectively:

$$L_{Gi} = P_{Gi} \cdot MLC_i^p \quad i=1, \dots, n \quad (34)$$

$$L_{Di} = P_{Di} \cdot MLC_i^o \quad i=1, \dots, n \quad (35)$$

G. Reconciled Marginal Allocation

As result of non-linearity of losses, the sum of allocated losses using the marginal allocation does not match with the actual losses of the system.

$$\sum_{i=1}^n L_{Gi} + \sum_{i=1}^n L_{Di} > L_{AC} \quad (36)$$

where

$$L_{AC} = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n G_{ij} [V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_i - \theta_j)] \quad (37)$$

Therefore, losses allocated must be normalized through a reconciliation factor k_o [1].

$$L_{Gi} = P_{Gi} \cdot k_o \cdot MLC_i^p \quad i=1, \dots, n \quad (38)$$

$$L_{Di} = -P_{Di} \cdot k_o \cdot MLC_i^o \quad i=1, \dots, n \quad (39)$$

where

$$k_o = \frac{L_{AC}}{\sum_{i=1}^n L_{Gi} + \sum_{i=1}^n L_{Di}} \quad (40)$$

3. Distributed Generation Penetration

The penetration of distributed generation into a given distribution system could be measured through an individual penetration index [11] associated with the generator i :

$$\eta_i = P_{Gi} / P_D \quad (41)$$

where:

P_{Di} Real power injected by generator i to the grid

P_D Maximum demand of the system

or through a general penetration index written as the sum of all individual penetration indexes:

$$\eta = \sum_{i=1}^n \eta_i = \sum_{i=1}^n P_{Gi} / P_D \quad (42)$$

At given point, if η is less than 1 distribution system requires power injections from grid supply point. If η is greater than 1 the distribution system exports power towards the transmission system.

4. Case Study

As shown in Fig. 1 a 28-bus 15kV distribution system is used to compare the seven distribution loss allocation procedures. The test system comprises 25 demands and two wind parks connected at bus 27 and 28 respectively. Each wind park has a generation capacity of 15MW. Due to regulatory requirements both wind parks must generate reactive power (using VAR compensation through capacitors, if required) in order to guarantee a voltage level between 0.95 and 1.05 pu in each own busbar [12].

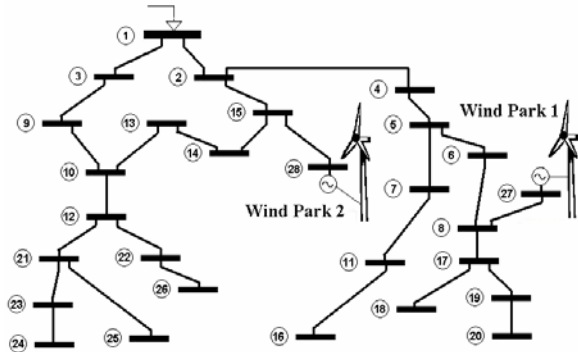


Fig. 1. 28-bus distribution system

At peak time total active and reactive demand are 15.5MW and 4.6MVar as shown in Table II of the appendix. Bus 1 is the grid supply point and represents the power interchange point with transmission system. Data line can be found in Table III of the appendix.

A. Avoided and produced losses as function of penetration indexes.

Fig. 2 shows the distribution system losses L produced in the system by the interaction of both wind parks with different penetration level.

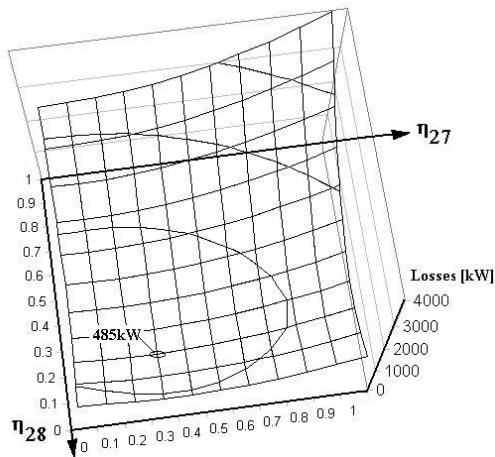


Fig. 2. Distribution system losses

The minimum power losses in the system (485kW) are reached when $\eta_{27} = 0.3$ ($P_{G27}=4.65$ MW) and $\eta_{28} = 0.2$ ($P_{G28}=3.10$ MW). This operation point also corresponds with the maximum avoided losses in the system (764kW) as seen in Fig. 3. The avoided losses of the system are defined as:

$$\text{Avoided Losses} = L_{\text{without DG}} - L_{\text{with DG}} \quad (43)$$

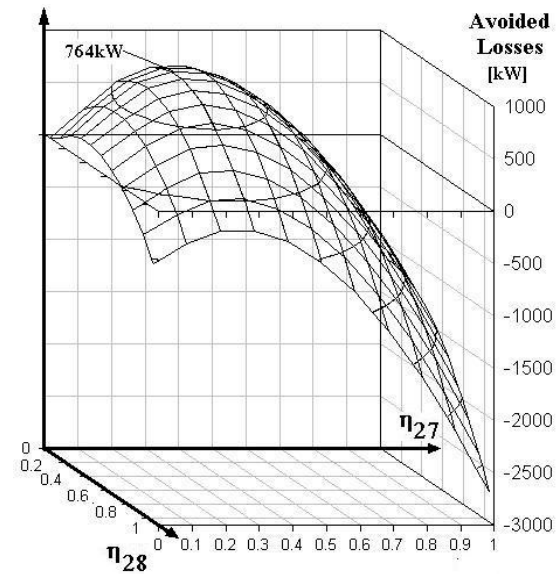


Fig. 3. Avoided distribution system losses

Fig. 4 shows a graph of avoided or produced losses as function of penetration indexes of distributed producers. It is important to point out that high penetration levels usually derive in an increase of power losses respect to a state without distributed generation.

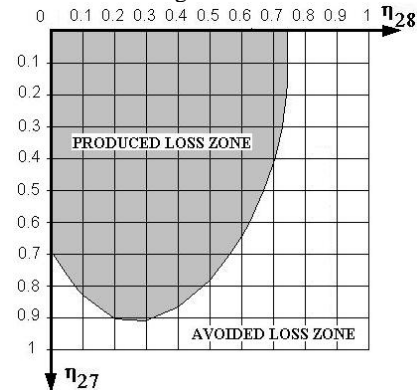


Fig. 4. Graph of avoided and produced distribution losses

B. Losses allocated as function of penetration indexes using seven allocating procedures.

Using the seven procedures described in detail in Section 2 we are able to build graphs of power losses allocated for each wind park (L_{G27} and L_{G28}) and all demands (L_{D2}, \dots, L_{D26}) as function of individual penetration indexes associated to both generators (η_{27} and η_{28}) described in Section 3.

Fig 5, 6 and 7 presents the results associated to wind park 1 (bus 27), wind park 2 (bus 28) and demand 11, respectively. Losses allocated are given in kW. Penetration indexes go from 0 to 1. This mean that power injected to grid can vary from 0MW to 15.5MW in each wind park.

The allocation of negative or positive losses must be interpreted under a cost allocation policy as follow: positive losses imply a charge or payment for produced losses and negative losses imply a reward for avoided losses.

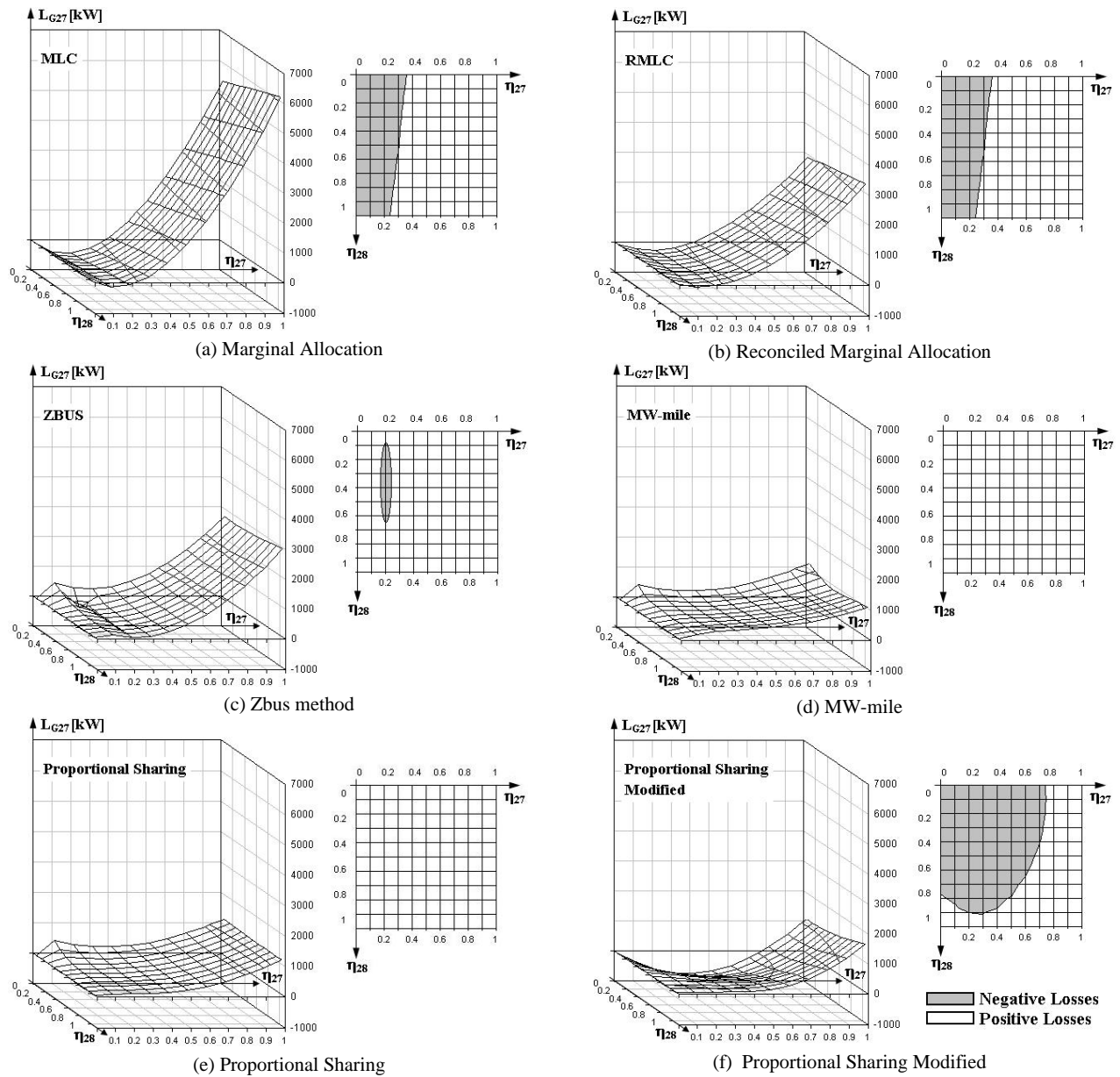


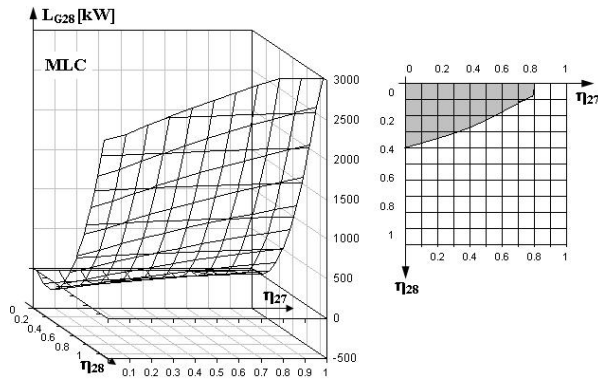
Fig. 5. Losses allocated to Wind Park 1 (bus 27)

In general, as seen in all graphs, it is confirmed that marginal procedures, circuit based and proportional sharing modified are able to assign positive and negative losses. On the other hand, postage stamp, MW mile and proportional sharing procedures always derive in positive losses. Also, is verified that all procedures recover the entire losses of the system except the marginal procedure that recovers approximately twice power losses. This result is obtained as consequence of non-linearity of losses justifying the application of a reconciled marginal procedure. Despite postage stamp, MW-mile and proportional sharing procedures were supported under different foundations, results shows similar behavior for both wind parks and all demands (see figs. 5d, 6d, 7d, 5e, 6e and 7e). In fact, postage stamp results were very similar to the obtained using MW-mile procedure. Then, postage stamp graphs were not included here due to economy of space.

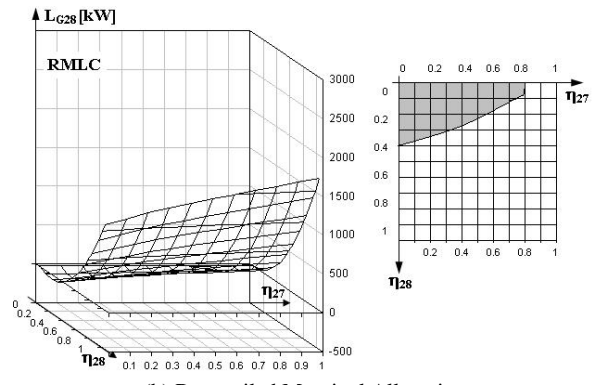
These results are meaningful because postage stamp procedure does not use network information and the other two: MW-mile and proportional sharing procedures are based on power flow solutions and network topology information. The similarity of results could be explained as consequence of the arbitrary assignation of 50%-50% among producers and demands.

If another arbitrary assignation is applied as proposed in proportional sharing modified (demand pays for 100% of losses and producers pays or receives for avoided or produced losses) the solutions are close to the achieved using reconciled marginal procedures (see figs. 5b, 6b, 7b, 5f, 6f and 7f).

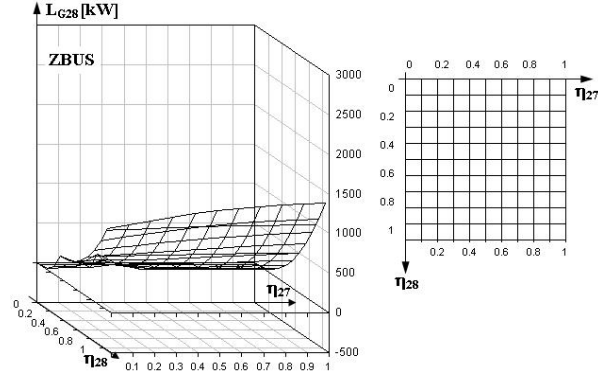
Another important fact to point out is that proportional sharing modified procedure allocates negative losses to producers in the same way that avoided losses were distributed as function of penetration indexes (see figs. 4, 5f and 6f)



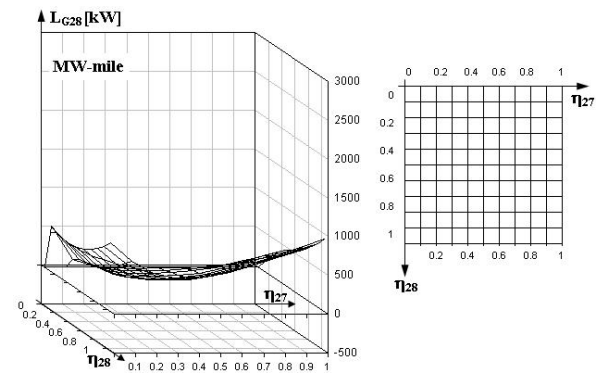
(a) Marginal Allocation



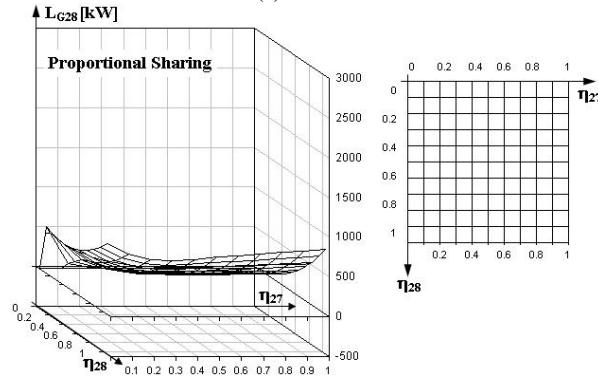
(b) Reconciled Marginal Allocation



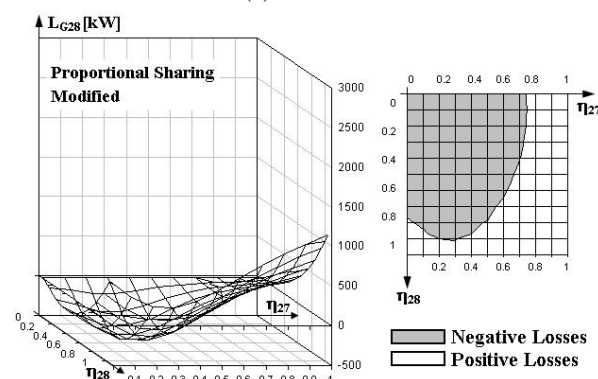
(c) Zbus method



(d) MW-mile



(e) Proportional Sharing



(f) Proportional Sharing Modified

Fig 6. Losses allocated to Wind Park 2 (bus 28)

Finally, it is observed that Zbus method achieves comparable results respect to reconciled marginal procedure for loss allocation to demands (see figs. 7b and 7c). It could be explained due to as marginal procedures the Zbus method is supported in the network equations and uses a non-arbitrary natural division of losses through Zbus matrix in a nodal basis. However, Zbus method requires an arbitrary assignment of 50%-50% in order to allocate losses among demands and producers.

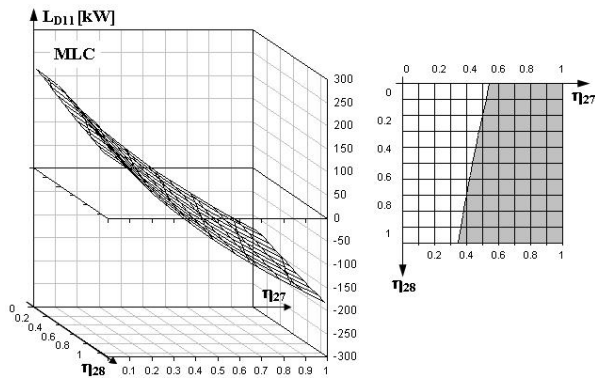
From the above results and particularly when a high penetration level is achieved ($\eta = \eta_{27} + \eta_{28} > 0.7$) it is worth noting the following results:

a) Marginal procedures reach high volatility. In fact the highest values for losses were achieved for loss allocation to producers (see figs 5a, 6a and 7a). For instance when $\eta > 0.7$, wind park 1 must pay at least 2180kW. If $\eta = 2$ wind park 1 must pay 6160kW

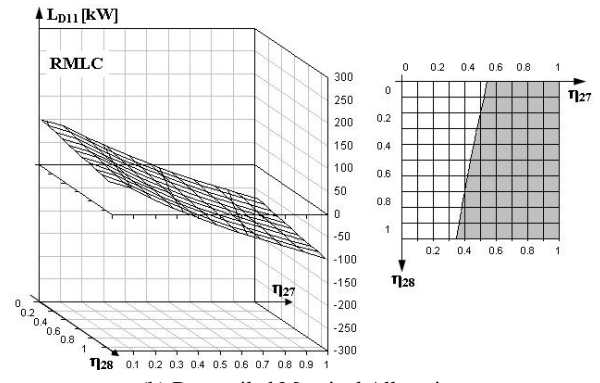
when and real losses at this point are 3965kW and demand 11 must receive 186kW by avoided losses.

b) Reconciled marginal procedures permits to mitigate some volatility (see figs 5b, 6b and 7b).. For instance, wind park 1 must pay at least for 1150kW when $\eta > 0.7$ and 3320kW when $\eta = 2$. On the other hand, Demand 11 receives not less than 100kW when $\eta > 0.7$ and 100kW when $\eta = 2$.

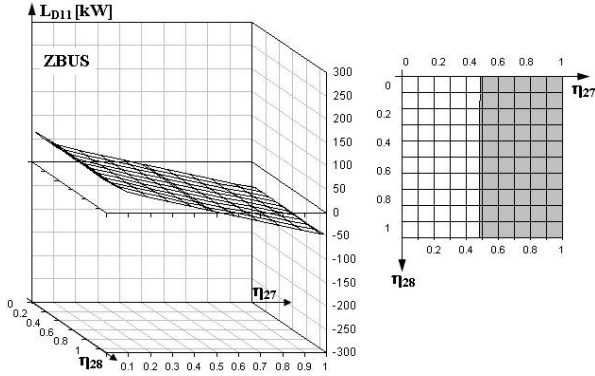
c) Under a marginal allocation procedure, reconciled or not, a high penetration of distributed generation derives in positive losses for producers and negative losses for demands. In general, this means that a very high penetration of renewable generation may produce that distributed generators have to pay for increase of losses and demands have to receive a compensation for avoided losses.



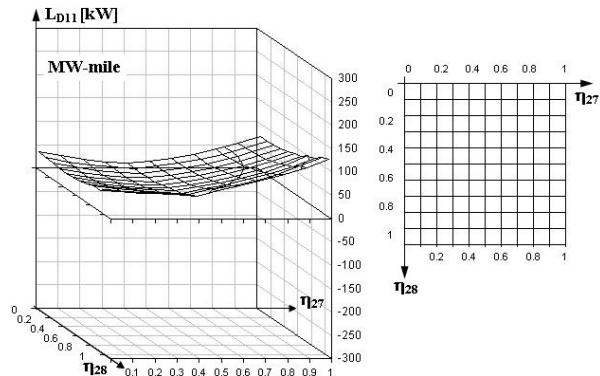
(a) Marginal Allocation



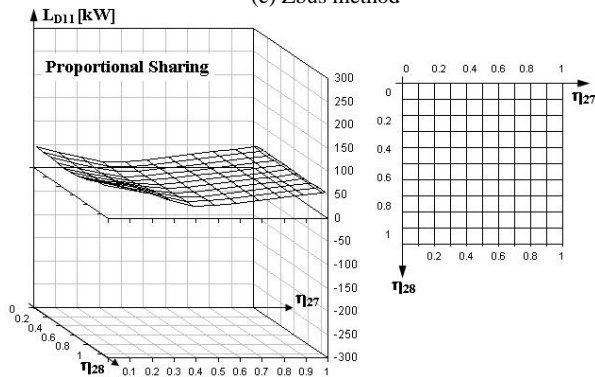
(b) Reconciled Marginal Allocation



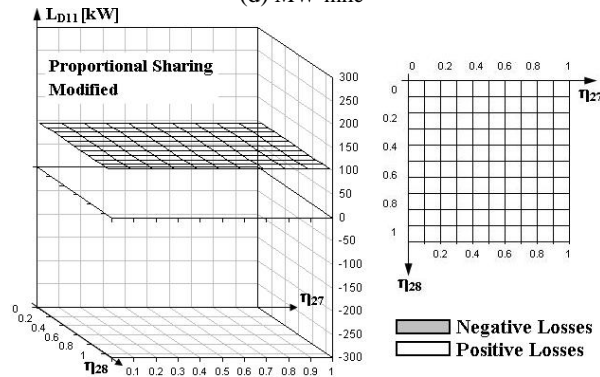
(c) Zbus method



(d) MW-mile



(e) Proportional Sharing



(f) Proportional Sharing Modified

Fig 7. Losses allocated to Demand 11

d) Loss allocation using postage stamp, MW-mile and proportional sharing procedures derive always in positive and moderate charges respect to marginal procedures. Table 1 shows losses allocated to both wind generators and demand 11 when $\eta = 2$.

Table I. - Losses allocated when $\eta = 2$

	Postage Stamp	MW-Mile	Proportional Sharing	Proportional Sharing Modified
L_{G27}	990kW	1045kW	1165kW	1596kW
L_{G28}	990kW	937kW	816kW	1118kW
L_{D11}	115kW	121kW	50kW	100kW

e) Loss allocation using a proportional sharing modified offers interesting results. As seen in fig 7f, losses allocated to demand are a fixed number obtained from his contribution disregarding the effect of distributed generators using the proportional sharing method.

Then, losses allocated to demands do not depend of penetration index η . On the other hand, losses allocated to generators are given by the effect of penetration of distributed generators into general avoided/produced losses as shown in fig 4. Table 1 shows losses allocated to both wind generators and demand 11 when $\eta = 2$.

Really, this procedure constitutes a hybrid model. The loss allocation process behaves on similar way as a marginal approach when the distributed generation is capable to avoid losses. Conversely, when generators are not capable to avoid losses, the procedure behaves standard proportional sharing procedure. For instance, if the generation penetration indexes are low, the distributed generators are capable of avoid losses. In these circumstances, generators are rewarded by avoided losses and demands are charged by produced losses. This behavior is like a marginal behavior reported above.

Conversely, when the generation penetration indexes are high distributed generators do not avoid losses and are charged by produced losses. However all demands still charged by produced losses.

5. Conclusions

This paper presents a comparative study of seven loss allocation procedures taking into account different levels of penetration of renewable generation in distribution networks. Results permits to conclude that a high penetration of distributed generation usually derives in an increase of total system losses respect to the total losses disregarding the effect of the generators. In consequence, if marginal loss allocation procedures are applied final result yields in volatile charges to be paid by generators and high compensations for demands. Other methods as postage stamp, MW-mile and proportional sharing are capable to diminish the volatility of marginal solutions bt means of the application of positive charges for all user of the network. A proportional sharing modified procedure constitutes a hybrid model behaving on similar way as a marginal approach when the penetration of distributed generation is low. Conversely, when the penetration of distributed generation is high, the procedure behaves as a standard proportional sharing method.

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Appendix

Table II. - Peak demand

Node	Load		Node	Load	
	kW	kVAr		kW	kVAr
1	0	0	14	600	100
2	350	100	15	850	300
3	700	260	16	850	150
4	700	280	17	450	100
5	560	150	18	600	200
6	180	70	19	800	300
7	600	240	20	900	250
8	650	260	21	350	100
9	650	100	22	630	250
10	800	300	23	750	100
11	900	120	24	560	200
12	450	120	25	450	180
13	600	200	26	570	200

Table III. - Data Line

Nodes		R	X	Bcap	Nodes		R	X	Bcap
From	To	(pu)	(pu)	(pu)	From	To	(pu)	(pu)	(pu)
1	2	0.2407	0.368	0.1	10	13	0.2097	0.3204	0.1
1	3	0.2745	0.4188	0.1	11	16	0.1541	0.2357	0
2	4	0.3472	0.5296	0	12	21	0.1808	0.2762	0
2	15	0.2407	0.368	0.1	12	22	0.2407	0.368	0
3	9	0.2576	0.3927	0.1	13	14	0.2097	0.3204	0.1
4	5	0.3472	0.5296	0	14	15	0.3835	0.2533	0.1
5	6	0.1808	0.2762	0	15	28	0.1808	0.276	0
5	7	0.2407	0.368	0	17	18	0.5987	0.396	0
6	8	0.2097	0.3204	0	17	19	0.6812	0.4508	0
7	11	0.2407	0.368	0	19	20	0.3835	0.2533	0
8	17	0.2745	0.4188	0	21	23	0.5987	0.396	0
8	27	0.2407	0.368	0	21	25	0.3835	0.2533	0
9	10	0.1541	0.2357	0.1	22	26	0.2407	0.368	0
10	12	0.2676	0.408	0	23	24	0.2745	0.4188	0

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