European Association for the Development of Renewable Energies, Environment and Power Quality

International Conference on Renewable Energies and Power Quality (ICREPQ'09)

Valencia (Spain), 15th to 17th April, 2009

CAPACITANCE EVALUATION ON PARALLEL-PLATE CAPACITORS BY MEANS OF FINITE ELEMENT ANALYSIS

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Abstract.

The electric field distribution produced by any disposition of insulating and conducting materials is a key aspect in electrical design, but exact values can only be obtained in simple geometries. In this work, using commercially available F.E.M. software we show the influence of the edge-effect on the electric field distribution of a two parallel-plane conducting plates system surrounded by an insulating medium taking into account the thickness of the conducting plates. We compare our results with previous published works. Finally, we obtain the relationship between capacitance and insulation characteristics, insulation gap, plate dimensions and plate thickness.

Key words

Modeling, Electric field, Capacitance, Finite Elements Method, Edge-effect.

1. Introduction

For several years, in the field of bioelectronics, flat conductors are being used to measure differences of voltage in the surface of the skin of the patients [1], with the aim of processing these signals [2;3] and thus being able to diagnose the pathologies without having to use invasive methods. The development of these technologies of noninvasive diagnosis supposes important economic savings in the sanitary system that benefits directly the whole society.

A field in particular in which there is an ongoing work is the development of this type of sensors, based on a flat conductor that is in contact with the skin. The measurement protocol used is based on positioning on the skin a reference sensor or reference electrode and to locate around it several other sensors or electrodes, so that the measured differences of potential are related to the reference electrode.

In the case of the intestinal pathologies, the signal that is registered with this type of electrodes, is called electroenterogram (EEnG) and is used to determine the intestinal mobility [3].

The intestinal mobility is generated by the intestinal myoelectrical activity, which consists of two signals: the slow wave (SW) and the Spike bursts (SB).

In recent years, the registry of the slow wave in surface has become the objective of numerous research groups. This signal is characterized by its low frequency, of the order of 0,3 Hertz in dogs and of 0,2 Hertz in humans. The main problem is that the frequency of the SW is very similar to the breathing frequency, which makes imperative the use of powerful signal processing algorithms that are in charge to separate both signals.

The registered differences of potential in the abdominal surface with this method are of the order of μV . Due to the small amplitude of the registered signal, it is necessary to use sensors with the minimum effect to the signal that is tried to register, that is to say, that reduce to the minimum the ratio signal interference (S/I).

The technological base of these sensors is the conversion of the electrical charge in an electrical potential. With this premise, the quality of a sensor is determined by the interference level that introduces in the charge distribution that it tries to measure. In this way, the ideal sensor is the one that does not introduce any interference.

Being sensors based on flat conductors located on the abdominal surface, that is a surface with certain dielectric permitivity, what is being generated is, in fact, a capacitor of flat conducting plates.

It is known that an important interference in the capacitance of a flat capacitor is the edge effect [4] , this interference is not despicable and directly affects the operation of the sensorial element since it modifies the capacitance of the set, because of the outer electric field lines. (see Fig. 1).

In this paper we study the distribution of the electric field and the capacitance of an electrical device constituted by two conductors of rectangular section and unitary length.

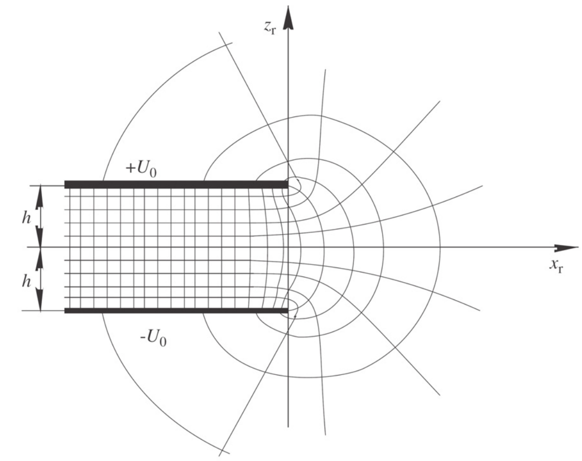

Fig. 1. Two-dimensional model of a parallel plate capacitor and its electric field distribution [4].

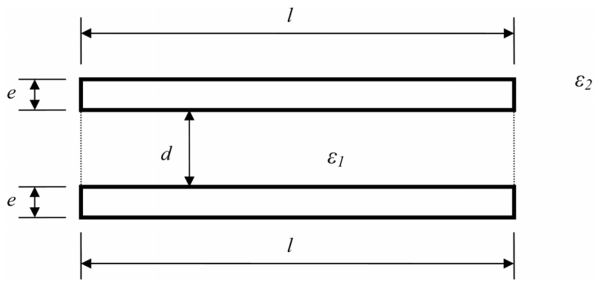

Fig. 2. Two-dimensional model of a plane capacitor.

The objective of this study is to formulate equations that allow knowing the real capacitance of the capacitor generated in applications where conductors of rectangular section are used.

Nowadays, owe to the calculation power of the computers, different iterative algorithms are used to solve the equations that govern the behaviour of this type of capacitors [5-9], these algorithms are designed to obtain the surface distributions of load, the values of the electric field and the total capacitance. The disadvantage of these algorithms is that they are designed for specific cases, so that, if geometry changes, the algorithm must almost be reconstructed entirely.

At present, due to the development of the method of the finite elements (FEM), commercial packages exist that integrate this technique and that tackle electrostatics problems of generic form, regardless of its geometry. This means not only increasing flexibility but also diminishing costs and times of development. This is because, in these packages, a graphical interface allows to analyze any geometry with the suitable contour conditions.

In this case we have used the commercial package software ANSYS [10] , since this commercial package has a specific module to solve electrostatics problems.

2. Parallel plane conductors

The most common capacitor in electrical engineering is formed by two parallel plates. This capacitor has been widely studied when its behaviour is ideal [11] , that is, when the edge effects are not considered [12].

With the aim of comparing a real capacitor with the ideal case, the commercial software has been used to calculate the total capacitance of the capacitor [13] and the distribution of the electric field between the plates [4]. To achieve this, a model of two parallel conductors of rectangular section has been generated, in this model all the geometric characteristics can be varied, as well as the relative permitivities.

In the Fig. 2 the geometric model is shown, where e is the thickness of the conductors, l is the width of the conducting plate, d is the distance between plates, ε_1 is the relative permitivity of the dielectric media placed between the plates and ε_2 is the relative permitivity of the dielectric material that surrounds the system.

In **Table I** the variables of the model and the values used for the simulation are shown.

Table I: Value of the variables.

l (m)	e (m)	d (m)	ε_1	ε_2
0.5	0.01	0.1	1	1
0.875	0.0325	0.325	251	1
1.25	0.055	0.55	500	1
1.62	0.0775	0.775	750	1
2	0.1	1	1000	1

The values have been chosen so that they agree with the values used previously by other authors to allow the comparison of results. Combining the values of the variables of Table I, 625 simulations have been realized.

In order to evaluate the capacitance of the ideal capacitor the classical equation has been used [11].

$$C_{ideal} = \varepsilon_0 \cdot \varepsilon_1 \cdot \frac{S}{d} = \varepsilon_0 \cdot \varepsilon_1 \cdot \frac{l}{d} \tag{1}$$

In Table II are shown the results of the simulation realized in a model with a length l=0.5m, a thickness e=0.01m and ε_2=1.

Table II: Calculation and modelling results.

l/d	ε_1	C_{IDEAL}	$C_{SIMULATION}$
5.00	2.51E+02	1.11E-08	1.39E-08
	5.00E+02	2.22E-08	2.77E-08
	7.50E+02	3.32E-08	4.15E-08
	1.00E+03	4.43E-08	5.53E-08
1.54	2.51E+02	3.42E-09	5.73E-09
	5.00E+02	6.82E-09	1.14E-08
	7.50E+02	1.02E-08	1.71E-08
	1.00E+03	1.36E-08	2.28E-08
0.909	2.51E+02	2.02E-09	4.15E-09
	5.00E+02	4.03E-09	8.26E-09
	7.50E+02	6.04E-09	1.24E-08
	1.00E+03	8.05E-09	1.65E-08
0.645	2.51E+02	1.43E-09	3.45E-09
	5.00E+02	2.86E-09	6.88E-09
	7.50E+02	4.29E-09	1.03E-08
	1.00E+03	5.71E-09	1.38E-08
0.50	2.51E+02	1.11E-09	3.06E-09
	5.00E+02	2.22E-09	6.09E-09
	7.50E+02	3.32E-09	9.13E-09
	1.00E+03	4.43E-09	1.22E-08

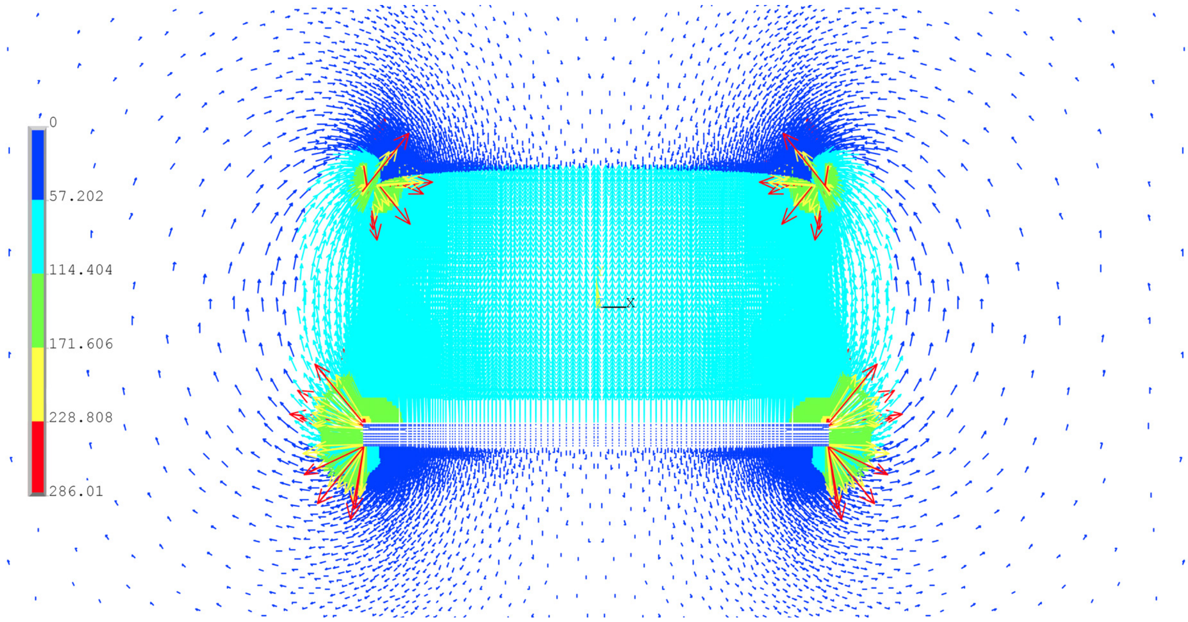

Fig. 3. Electric field distribution in plane capacitor.

A preliminary analysis of the values from **Table II** shows that C simulation /C_{ideal} does not depend on the relative permitivity of the dielectric located between the plates but. solely. on the separation between plates. the length of the plates and its thickness.

A detailed statistical analysis of the 625 calculated values shows that the best dependancy is obtained with the quotients *d/e* (*separation/thickness*) or *l/e* (*length/thickness*). The relation of the quotient of capacities to the ratio *separation/thickness* is slightly parabolic. almost linear. The relation of the quotient of capacities with the variable *distance/thickness* is hyperbolic. The simplest expression (2) that adjusts simulated to ideal values is:

$$\frac{C_{sim}}{C_{ide}} = 1 + k_1 \cdot \frac{d}{l} \tag{2}$$

Where k_1 = 1.01344. With this adjustment R^2 is = 97.47%.

Even a better adjustment is obtained with the parabolic dependancy (3) of the *separation/thickness* (R^2=98.16):

$$\frac{C_{sim}}{C_{ide}} = 1 + \frac{k_1 \cdot d + k_2 \cdot d^2}{l} \tag{3}$$

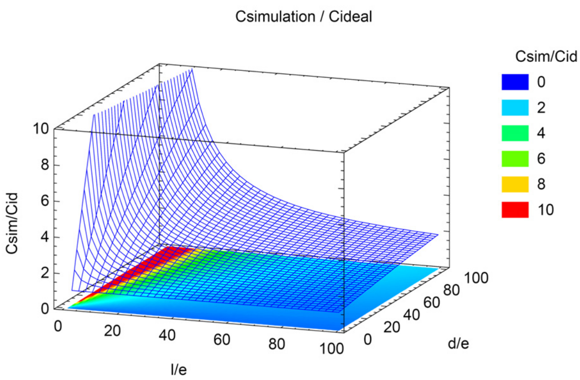

Fig. 4. $C_{simulation}/C_{ideal}$ related to *distance/thickness* and *length/thickness*.

Where k_1 = 1.06109 y k_2 = 1.6798E-03. With this adjustment R^2 is = 98.16 %.

Increasing the degree of the hyperbolic dependancy with the distance/thickness $R^2 > 99.61\%$ values can be obtained.

In Fig. 4 it can be observed that the real capacitance grows very quickly with respect to the theoretical capacitance when the variable length/thickness is reduced and. to a lesser extent. when the variable separation/thickness increases.

3. Electric field

In the realized simulations. the distribution of the lines of electric field around the conductors has been obtained.

Fig. 3 shows the vectorial form of the distribution of the electric field lines in a flat capacitor.

In Fig. 5 is shown a detail of the plate edge on a flat capacitor with the distribution of the electric field lines.

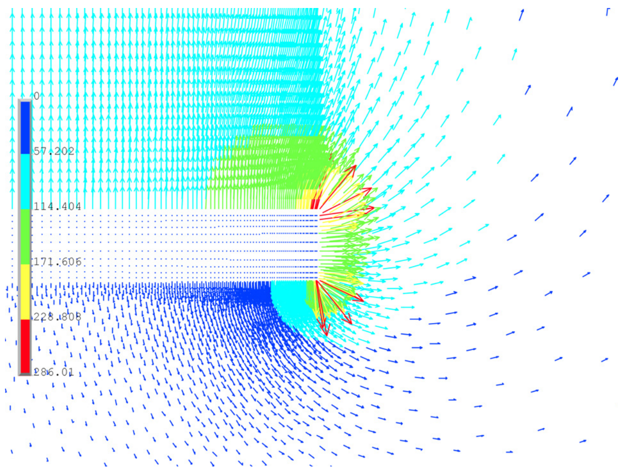

Fig. 5. Detail of the electric field distribution.

$$C_{Xiang} = -\frac{\varepsilon_0}{\pi}\cdot\left[\ln\lambda + \ln\left(1+2\cdot\lambda^{4}+15\cdot\lambda^{8}+150\cdot\lambda^{34}+\cdots\right)\right] \quad (4)$$

As can be seen in Fig. 3 and in Fig. 5. the value of the electric field is greater in the edges of the plates and. in addition. electric field lines exist in both surfaces of the plates. which agrees with the hypotheses given in [4].

4. Validation of the model

The validation of the model has been realized comparing the obtained values of the capacitance in the simulation with values of capacitance obtained applying the equation (4). This equation was proposed by Xiang in [13]. and is valid when the relative permitivity of the dielectric material placed between the conductors is ε_1=1. Due to this condition. for comparative purposes. only the simulation results with this permittivity have been taking into account.

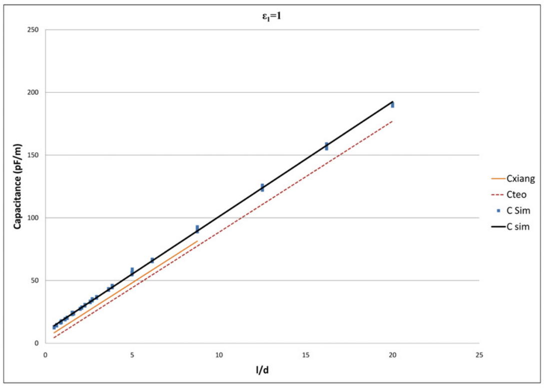

Fig. 6. FEM simulated capacitances vs. Y. Xiang capacitance.

In Fig. 6 is observed that the capacitance calculated by the model based on finite elements is in accordance with the capacitance calculated by Xiang while the ratio $l/d > 8$. for lower ratios the formula described by Xiang (4) gives lower capacitance values.

The λ coefficient from Xiang equation (4) is calculated with the following expression:

$$\lambda = \frac{1-\sqrt{k'}}{2\cdot\left(1+\sqrt{k'}\right)} = \frac{1-\sqrt{\tanh\frac{\pi\cdot l}{2\cdot h}}}{2\cdot\left(1+\sqrt{\tanh\frac{\pi\cdot l}{2\cdot h}}\right)} \quad (5)$$

Fig. 6 shows that the values obtained by Xiang for flat condensers of null thickness are higher than the values of the ideal classic expression. In our simulation we have verified that the real values. considering the real thickness from the plates. are even higher because of the plate thickness.

5. Conclusions

In this paper the validity of the method of the finite elements has been demonstrated to calculate the existing capacitance between two conducting plates. Firstly. the capacitance between two parallel conductors has been calculated and an expression has been obtained that allows to calculate the real capacitance between two mounted conductors of rectangular section.

It has been observed that in electrodes of small dimensions or with high plate to plate distance. the thickness of the plates dramatically modifies the value of the capacitance compared to an ideal capacitor.

It has been shown that it is possible to calculate the capacitance of any system of conductors for the industrial elaboration of multiconductive complex systems. that are the most frequent cases in the field of hybrid microelectronics.

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